Pipes in the Sky

Resource Allocation in Wireless Networks

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Wireless Networking & Communications Group

THE UNIVERSITY OF TEXAS AT AUSTIN

Outline

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How do 4 musicians play together?
such that you can hear what each one is playing...

The Problem

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- 4 musicians
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- Should be heard independently

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- 4 musicians
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First Idea - Soloing

Make the musicians take turns.

How do 4 musicians play together?

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Second Idea - Different Voices

Allot each musician a different 'voice'.

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- Different Voices + Soloing

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The Solution

- Different Voices + Soloing
- = string quartet.

Fine, but what about wireless communications?

The Problem

- How do n people simultaneously communicate.

The Solution

Fine, but what about wireless communications?

The Problem

- How do n people simultaneously communicate.
- Use techniques of wired communications.

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- Wireless channel $\longrightarrow$ 'resources'.

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- Wireless channel $\longrightarrow$ 'resources'.
- Resources $\longrightarrow$ 'orthogonal pipes'.

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The Solution

- Wireless channel → 'resources'.
- Resources → 'orthogonal pipes'.
- Intelligent resource allocation

What are these resources?

Primary Channel Resources

- Time

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- (Space)

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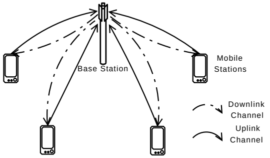

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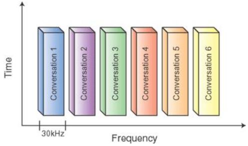

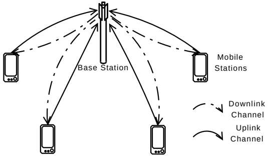

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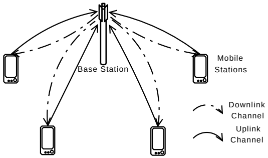

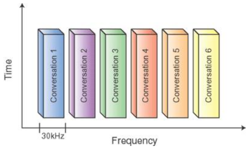

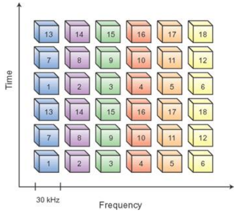

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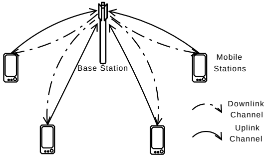

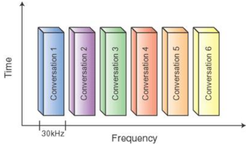

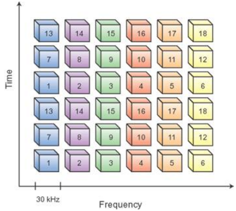

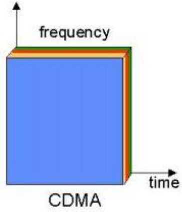

The stochastic nature of the wireless channel

Sources of Randomness

- Random channel coefficents

The stochastic nature of the wireless channel

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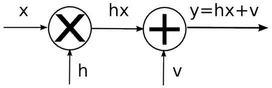

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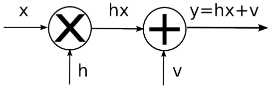

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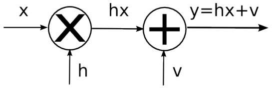

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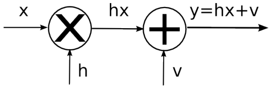

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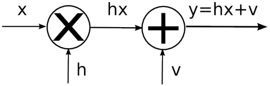

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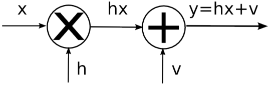

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- The channel coefficient in a time slot can be estimated.

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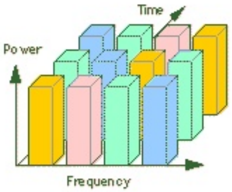

Intelligent Wireless Resource Allocation

What factors motivate intelligent resource allocation?

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Problem Decomposition

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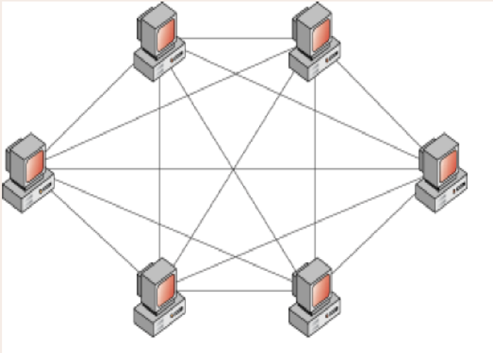
- Global problem: Optimal choice of local objective function.

Outline

The Scheduling Problem for General Networks

How do we schedule on a network with-

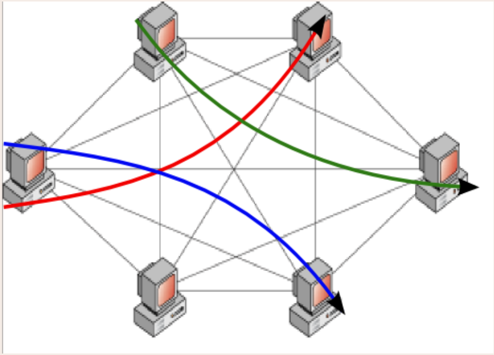
A General Communications System



The Scheduling Problem for General Networks

How do we schedule on a network with-

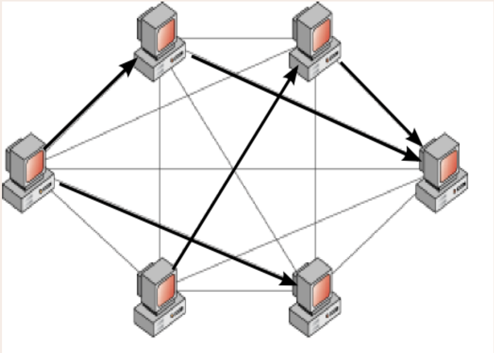
Multiple Input Flows



The Scheduling Problem for General Networks

How do we schedule on a network with-

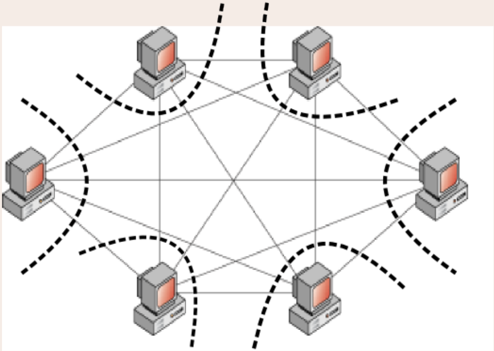
Random Communication Channels



The Scheduling Problem for General Networks

How do we schedule on a network with-

Constraints on Simultaneous Active Links



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- General topology
- Multiple flows
- Random channels
- Simultaneous transmission constraints

Aim of Scheduling Algorithm

- Given an input flow vector, stabilize queues

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The Back-Pressure Algorithm (BP)- An Overview

- Input: Current channel states ($h_{ij}[t]$) and queue states ($Q_i^d[t]$) in time slot t.

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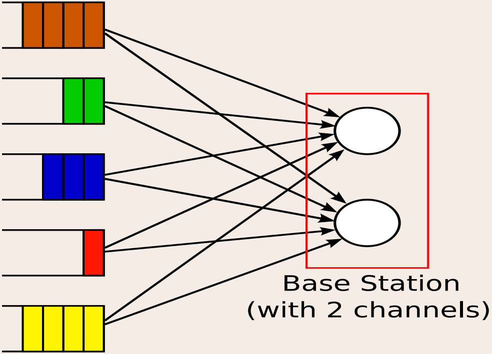
- Per-time slot optimization $\rightarrow$ Long-term stability.
- Input flow rates 'learnt' from queues.

Example 1: Uplink Scheduling

The System



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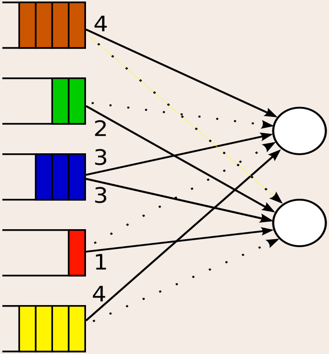
BP Algorithm- Stage 1



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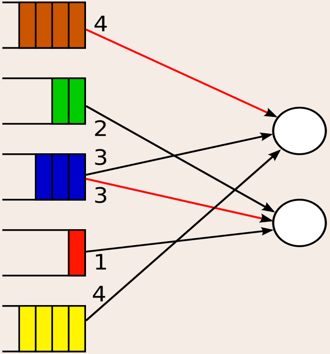
BP Algorithm- Stage 2



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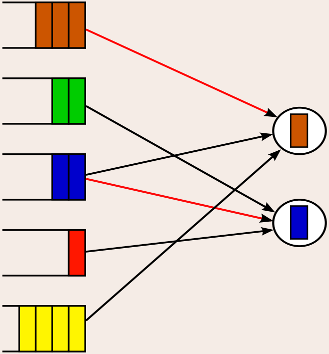
BP Algorithm- Stage 3



Example 2: Relay-aided Communications

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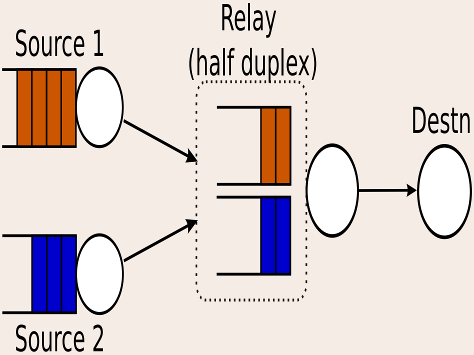

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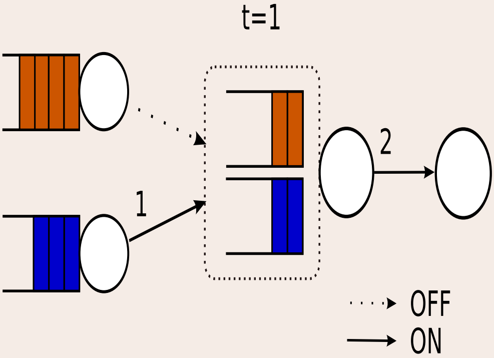

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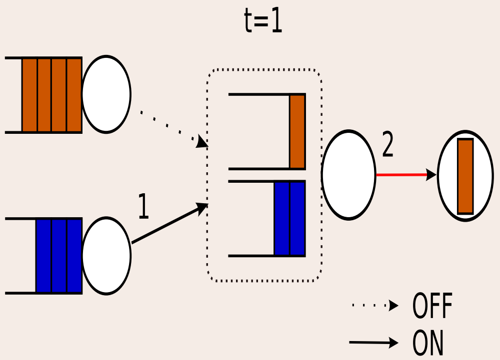

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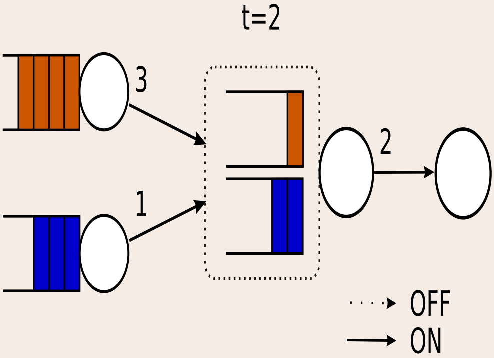

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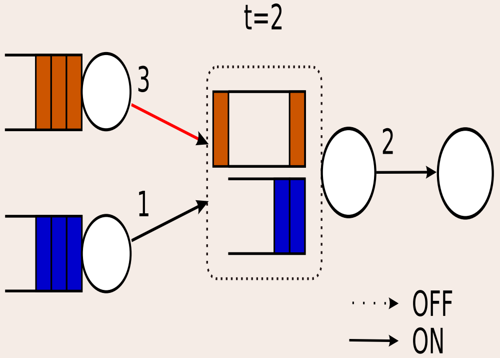

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$$C^*[t] = \arg \max_{C \subseteq V, C \in \mathcal{C}} \sum_{(i,j) \in C} w_{ij} h_{ij}$$

where $\mathcal{C}$ is the set of feasible simultaneous transmissions.

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- Stage 3: Activate all edges $e_{ij} \in C^*[t]$ and drain $Q_i^{d_{ij}^*}[t]$

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- Local objective $\equiv$ Max weighted sum of rates
- Need perfect CSI, feedback (of queue states).

Outline

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- w_i function of QOS metrics of user i (E.g. Throughput T_i, HOL packet delay D_i, queue length Q_i, etc.)

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