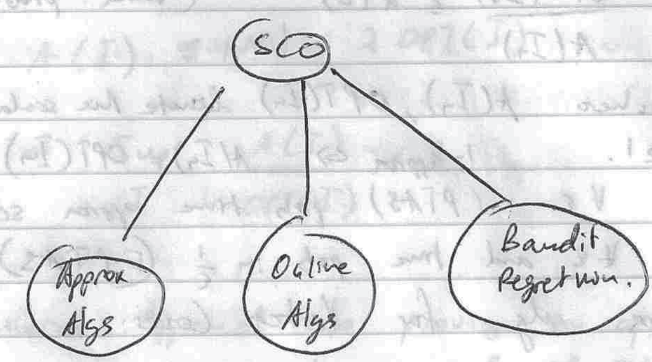
Lecture 1:

- Go over Syllabus
- Describe project + answer questions
- Describe next Due date

- Overview of Lectures



- Background Review: poly-time, poly-size w.r.t. length of input spec.

NP: Set of decision problems (Yes/No) with polynomial-sized certificates/witnesses that can be verified in polynomial time.

egs: 3-SAT,

We will talk about optimization problems.

- How to turn an optimization problem into a poly-time equivalent decision problem?

eg: Vertex cover

Appendix A.

Approx Alg defn: A is an α-approx alg. if

$\alpha(|I|)$

$A(I_n) \le \alpha(n)\, OPT(I_n)$

$$\sup_{I_n} \frac{A(I_n)}{OPT(I_n)} \le \alpha(n) \quad \text{(min problems)}$$

$\alpha(|I|)$

$A(I^*) \ge \cancel{\alpha(n)}\, OPT(I_n)$

I_n - instances of size n.

$$\sup_{I_n} \frac{OPT(I_n)}{A(I_n)} \le \alpha(n) \quad \text{(max problems)}$$

where $A(I_n)$, $OPT(I_n)$ denote the scalar values.

$\Rightarrow \alpha \ge 1$. 1-approx $\Leftrightarrow A(I_n) = OPT(I_n)$

$\alpha = 1+\varepsilon$ $\forall \varepsilon$ (PTAS) poly time approx scheme

$\alpha = 1+\varepsilon$ $\forall \varepsilon$ and time poly in $\frac{1}{\varepsilon}$ (FPTAS).

eg: Approx alg. for Vertex Cover:

while $\exists$ edges:

Alg: • pick any edge, add u,v to vert cover
• remove all edges incident to u,v.

Claim: Alg is a 2-approx. (best known for vert. cover)

pf. ~~$OPT(I) \le A(I)$ since $A(I)$ returns a vertex cover.~~

• ~~$A(I)$ returns a vertex cover~~
only remove edges after they are covered.

Alt. view: • find maximal matching M.
• include the vertices in M.

(one that cannot be made bigger by adding another edge) pf: • $A(I)$ returns a vertex cover.

• Claim: $A(I) \leq 2\, OPT(I)$

pf: • OPT must cover all the edges in M.

$$\frac{|M|}{2} \leq OPT(I)$$

$$A(I) = |M| \leq 2\, OPT(I)$$

$$\Rightarrow \sup_I \frac{A(I)}{OPT(I)} \leq 2$$

• General Approach to proving an approximation alg. is through a lower bound (for min probs)

• Not all probs have approx algs.

• General TSP: Find a min-cost tour (visiting each city exactly once)

No approx alg unless $NP = P$.

• Reduction from Hamiltonian Cycle.

• assume $\exists\ \alpha$ - approx. ff

• make every edge have length 1. non-edge have length $\geq n \cdot \alpha$

$\leq \alpha \cdot n$

• if result returned by approx, ham-cycle exists

else it does not exist

$> \alpha \cdot n$

- General Approach to inapprox:
  - assume approx exists.
  - show you can solve a different NP-complete problem with it.

- Approx for metric TSP:
  - in metric TSP, distances obey extra properties (metric).

    1) $d(u,u)=0$

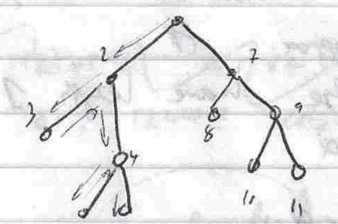
    2) $d(u,v) = d(v,u)$

    (triangle) 3) $d(u,v) + d(v,w) \geq d(u,w)$

Alg: • Find MST.
• pre-order traverse the MST. to output a tour.

Claim: Alg is 2-approx for metric TSP

pf: $A(I) \leq 2 \cdot \cancel{MST}.\text{cost}(MST)$



idea: preorder

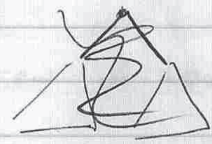
pf ~~by ind. on size of MST.~~

By triangle ineq.
direct path ≤ any other path.

~~step=1~~

~~...~~ ~~tour~~

~~step=k~~



- sum of paths on MST is ~~2EST~~ 2 cost(MST)

$\Rightarrow \quad A(I) \le 2\,\mathrm{cost}(MST)$

- $\mathrm{cost}(MST) \le OPT(I)$

pf: take tour, remove any edge to get an MST.

$\Rightarrow$ ~~...~~ $A(I) \le 2\,\mathrm{cost}(MST) \le 2\,OPT(I)$.

- Can further improve to Euclidean TSP which as PTAS.