Preventive Maintenance Models for Systems with Interactions

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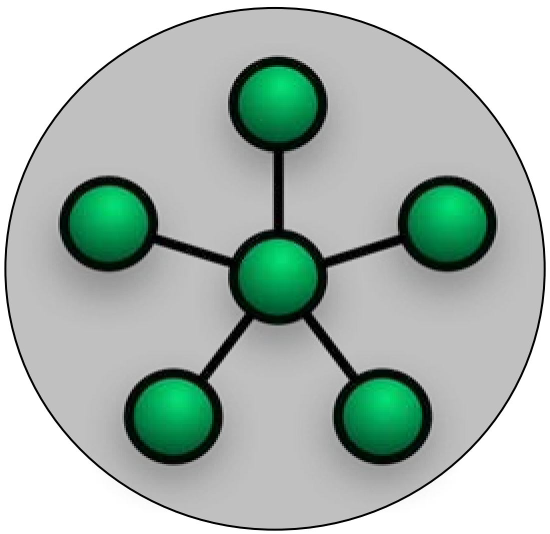
Facts

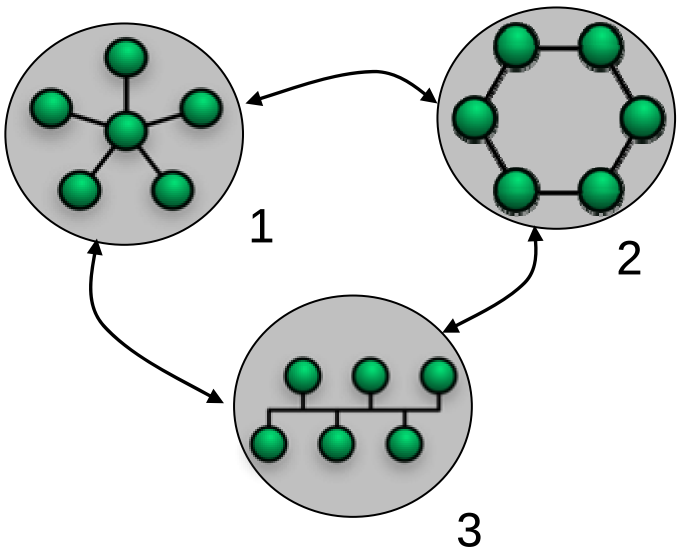
- Systems Deteriorate
- Failures can be Fatal
- Preventive Maintenance Helps

Example – A Computer Network



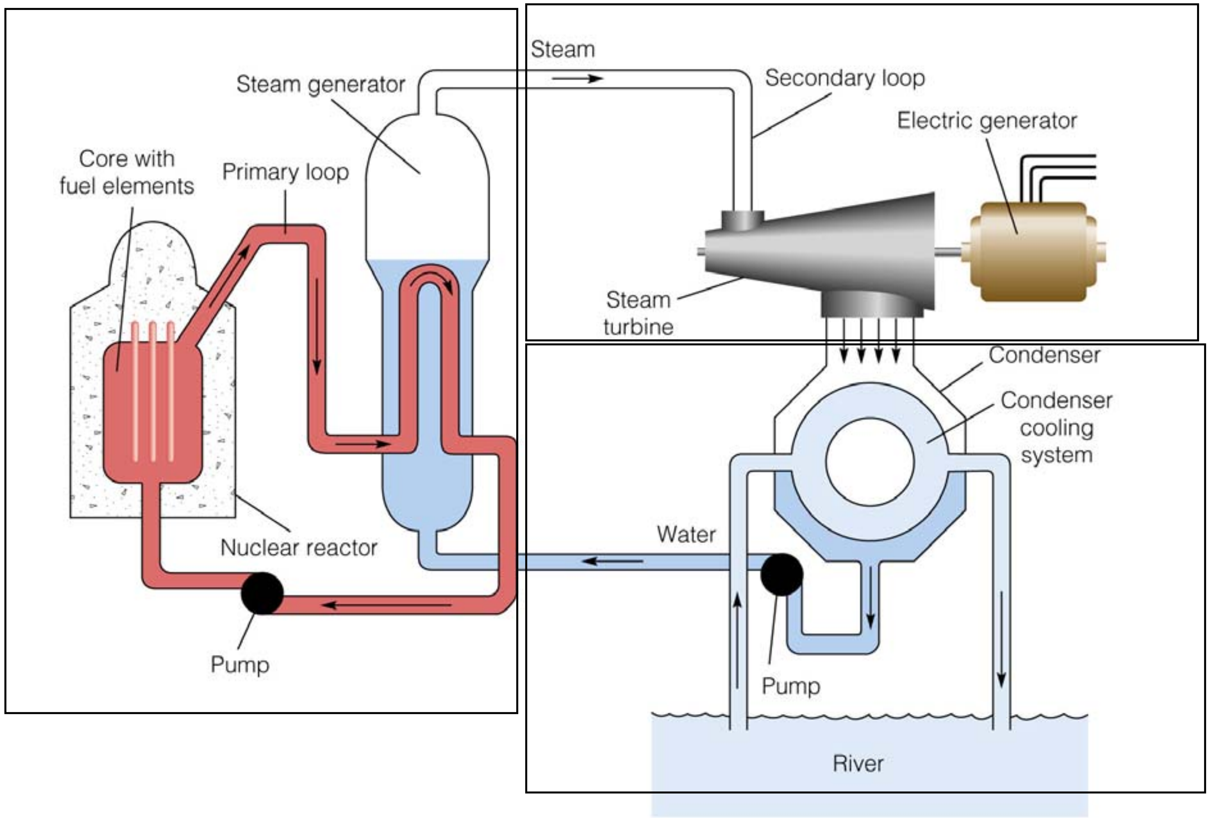
- Single System
- Failure Modes
- Minimal Repairs
- Periodic Replacements
- Triggers Failures

Example – A Network of Computer Networks



- Multiple Systems
- Failure Modes
- Minimal Repairs
- Periodic Replacements
- Interactions

Example – Systems of a NPP



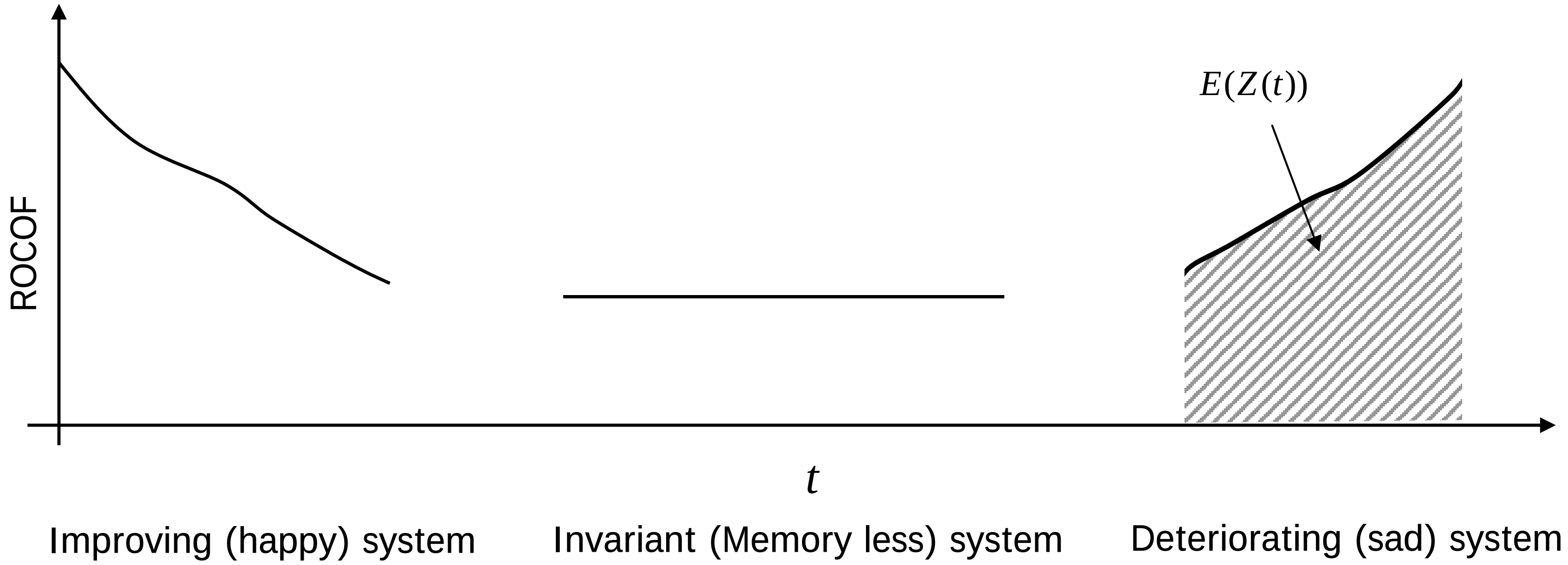
Outline

- Review
- Things to Model
- PM Model for a Single System
- PM Model for Multiple Systems
- Summary

Rate of Occurrence of Failures (ROCOF)

- ROCOF $= \frac{d}{dt}E(N(t))$

$$= \text{Instantaneous rate of average number of failures}$$

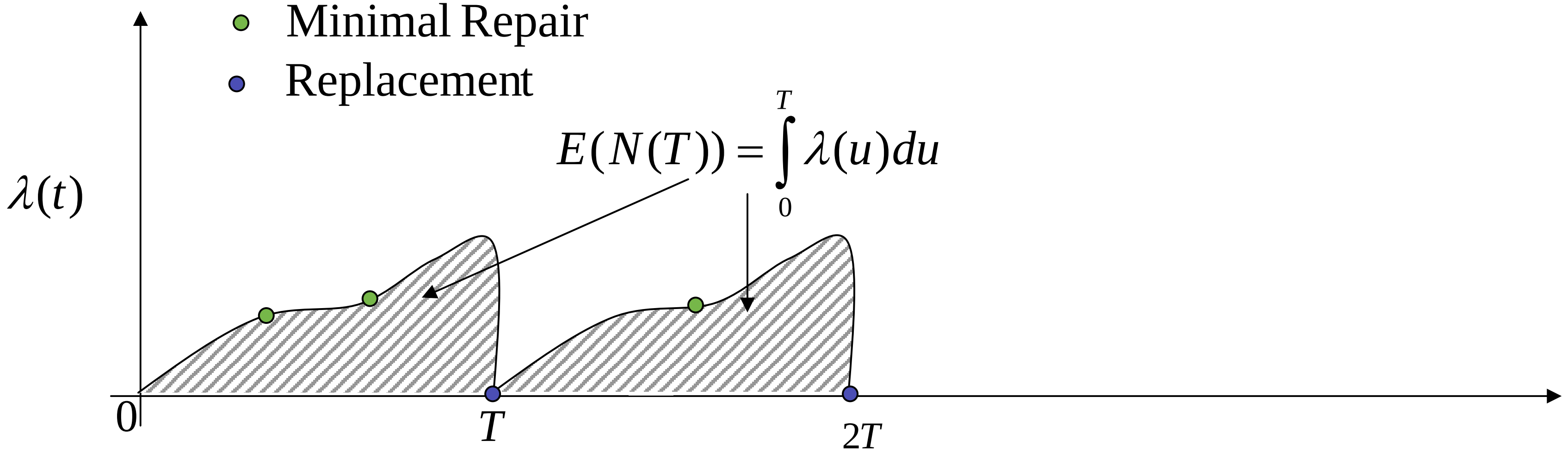


Non-Homogenous Poisson Process (NHPP)

- Counting Process $\{N(t), t \geq 0\}$
- Intensity Function $\lambda(t),\ t \geq 0$
- $N(t) \sim \text{Poisson}\,(\Lambda(t))$
- $\Lambda(t) = \int_0^t \lambda(u)\,du$ Cumulative Intensity Function
- $P(N(t) = n) = \dfrac{[\Lambda(t)]^n}{n!} e^{-[\Lambda(t)]} \quad n = 0, 1, 2, \ldots.$
- $E[N(t)] = \Lambda(t) \quad \Rightarrow \dfrac{d}{dt} E[N(t)] = \lambda(t) = \text{ROCOF}$

Minimal Repair Results in NHPP

- $N(t)$ is a NHPP[1] with intensity function $\lambda(t)$

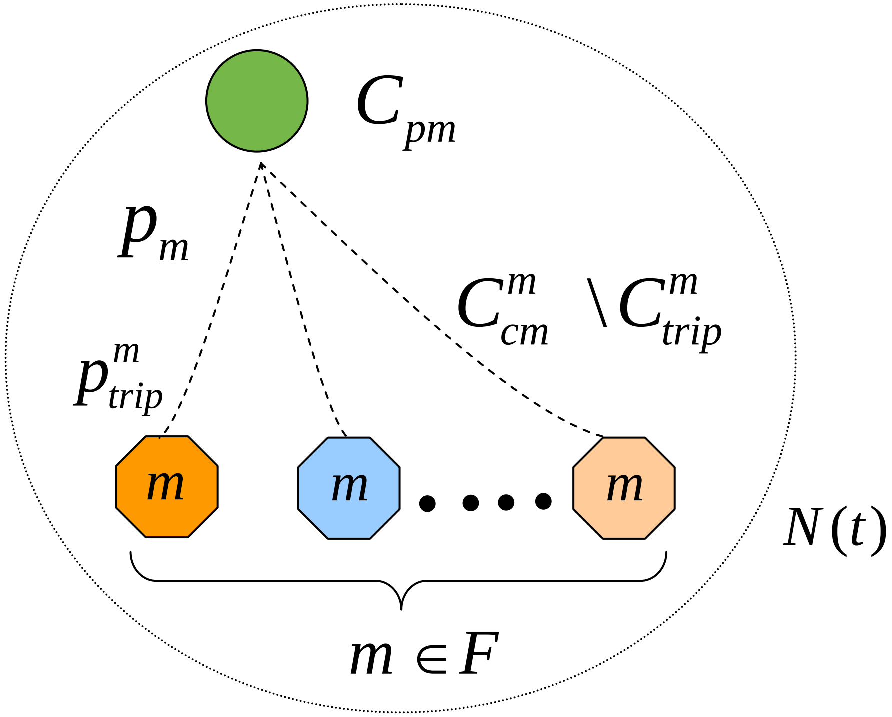


[1] R. Barlow and L. Hunter. Optimum preventive maintenance policies. *Operations Research* 8,90–100, 1960

Things to Model

- Maintenance Actions and Costs
  - Preventive (“as good as new”)
  - Corrective or Minimal (“as good as old”)
- System Failures
  - Natural
  - Induced
- Failure Modes
- Interactions
  - Failure
  - Structural
- Budget
- Finite Planning Horizon
- PM at the End

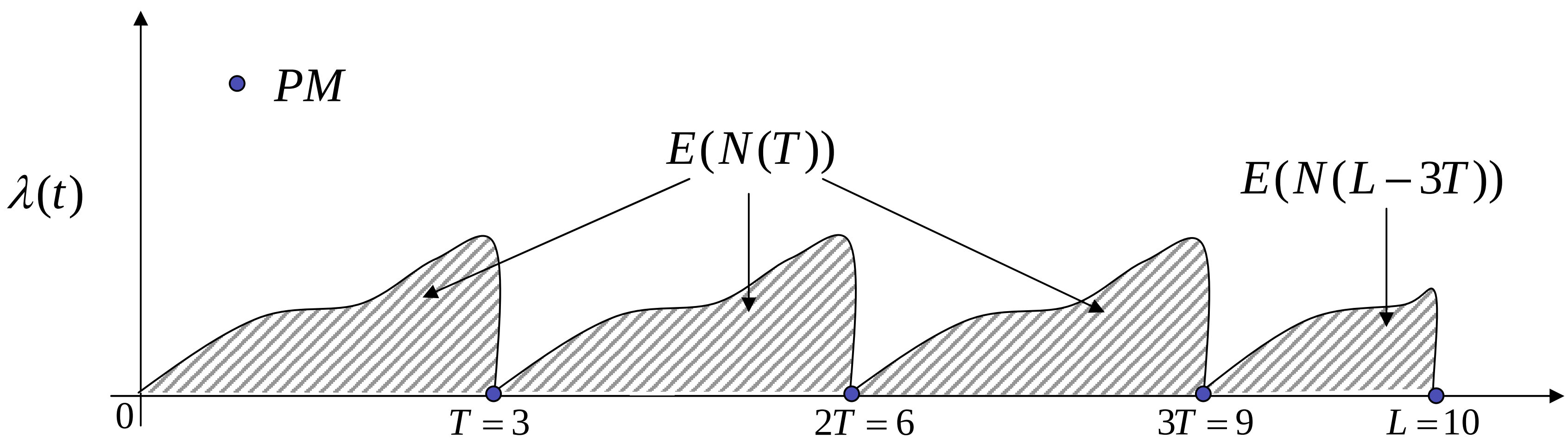
Preventive Maintenance Model for a Single System



Preventive Maintenance Model for a Single System

- Cost of PM = $C_{pm}\left\lceil \frac{L}{T} \right\rceil$
- Expected Cost of CM = $\left(\left\lfloor \frac{L}{T} \right\rfloor E[N(T)] + E[N(L - T\left\lfloor \frac{L}{T} \right\rfloor)] \right) \sum_{m \in F} \left(p_{trip}^{m} C_{trip}^{m} + (1 - p_{trip}^{m}) C_{cm}^{m} \right)$

$$\min_{T} C_{pm}\left\lceil \frac{L}{T} \right\rceil + \left(\left\lfloor \frac{L}{T} \right\rfloor E[N(T)] + E[N(L - T\left\lfloor \frac{L}{T} \right\rfloor)] \right) \overline{C}_{cm}$$



Preventive Maintenance Model for a Single System with Increasing ROCOF[2]

$$\min_T C(T)$$

$$C(T) = C_{pm}\left\lceil \frac{L}{T} \right\rceil + \left(\left\lfloor \frac{L}{T} \right\rfloor E[N(T)] + E[N(L - T\left\lfloor \frac{L}{T} \right\rfloor)] \right) \overline{C}_{cm}$$

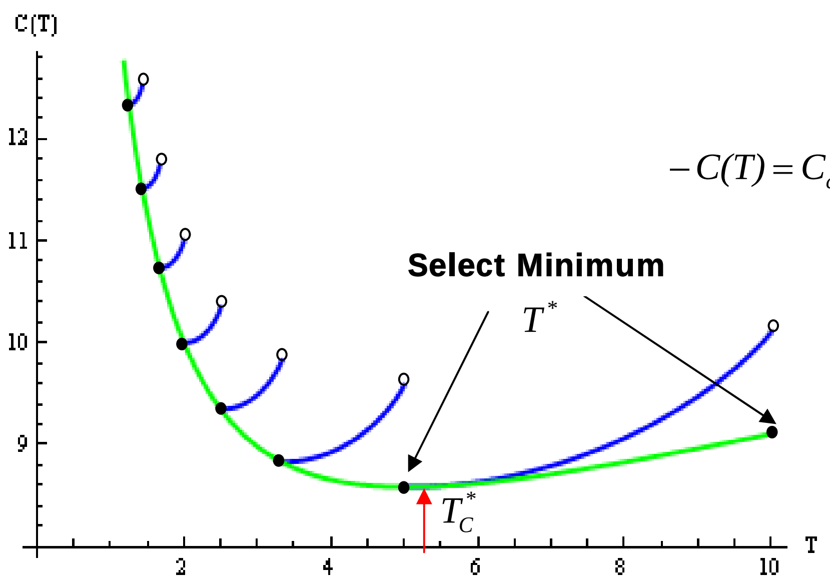
$$\min_T C_c(T)$$

$$C_c(T) = C_{pm}\frac{L}{T} + \left(\frac{L}{T} E[N(T)] \right) \overline{C}_{cm}$$

- $T_C^* = \arg\min_{T \in [0,L]} C_C(T)$

- $T^* = \arg\min_{T \in \{\frac{L}{n^*+1}, \frac{L}{n^*}\}} C_C(T)$

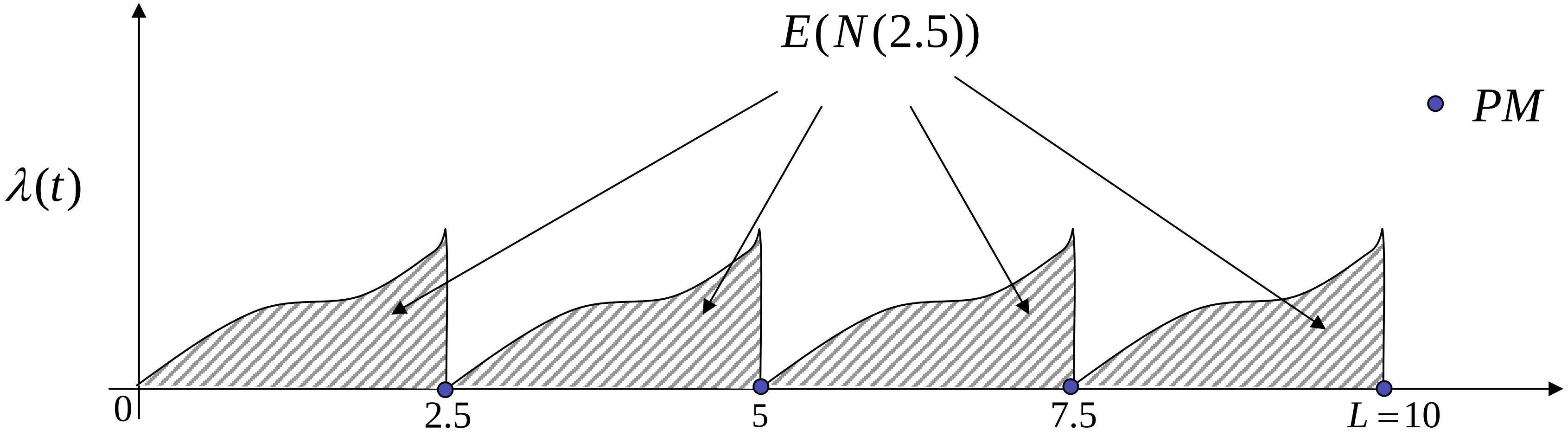
- n^* satisfies $\frac{L}{n^*+1} \le T_C^* \le \frac{L}{n^*}$



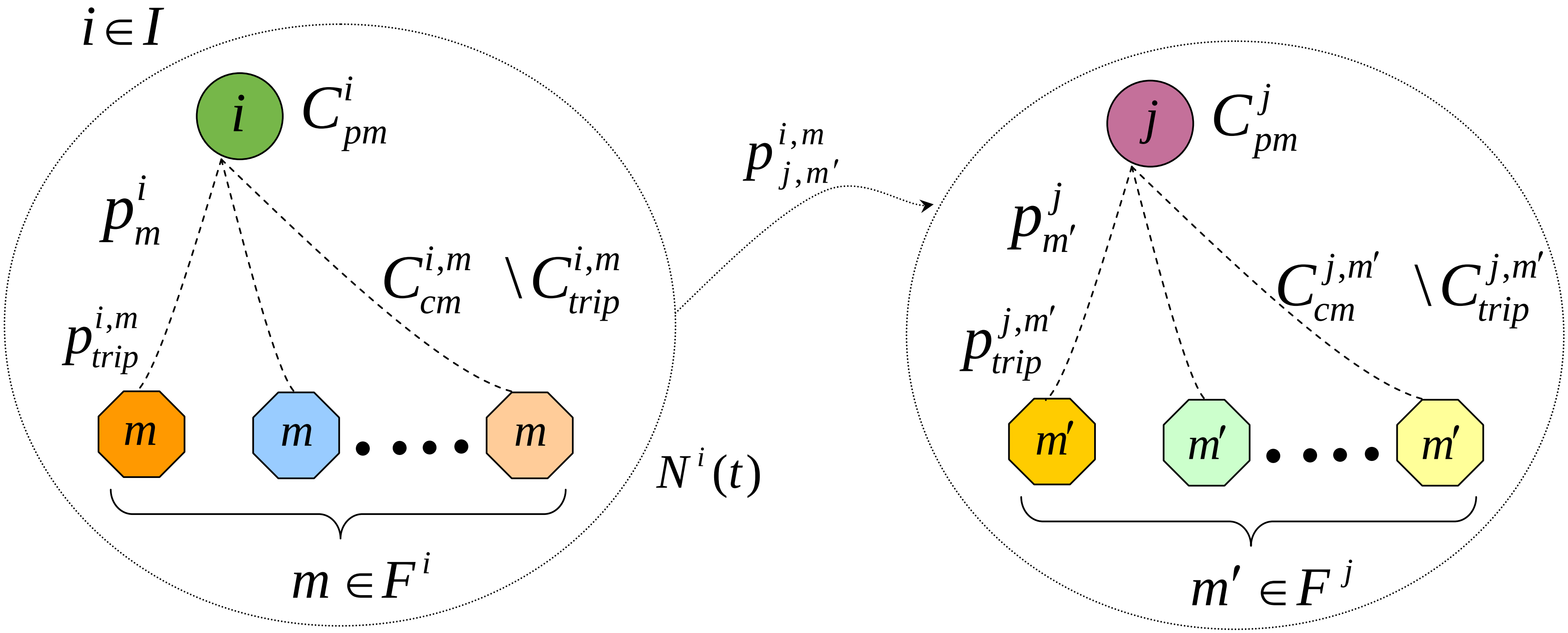
- $C(T) = C_c(T)$ at $\frac{L}{n}$, $n = 1,2,3,....$

[2] A. Glenko, D. Morton, E. Popova, R. Grantom, E. Kee, D. Richards and A.Sun, "Operational Models and Methods for Risk Informed Nuclear Asset Management" . In Proceedings Of The American Nuclear Society International Topical Meeting On Probabilistic Safety Assessment, American Nuclear Society International Topical Meeting On Probabilistic Safety Assessment, 2005

Optimal Solution for a Single System Preventive Maintenance Model with Increasing ROCOF



Preventive Maintenance Model for Multiple Systems



j is structurally dependent on i

Preventive Maintenance Model for Multiple Systems

$$\min_{\alpha_{ij},\,T_i} \sum_{i\in I}\left[\left(\left\lfloor \frac{L}{T^i} \right\rfloor E[N^i(T_i)] + E[N^i(L - T_i\left\lfloor \frac{L}{T_i} \right\rfloor)]\right)\overline{C}_i\right]$$

Subject to $$\sum_{i\in I} C^i_{pm}\left\lceil \frac{L}{T_i} \right\rceil \le B$$

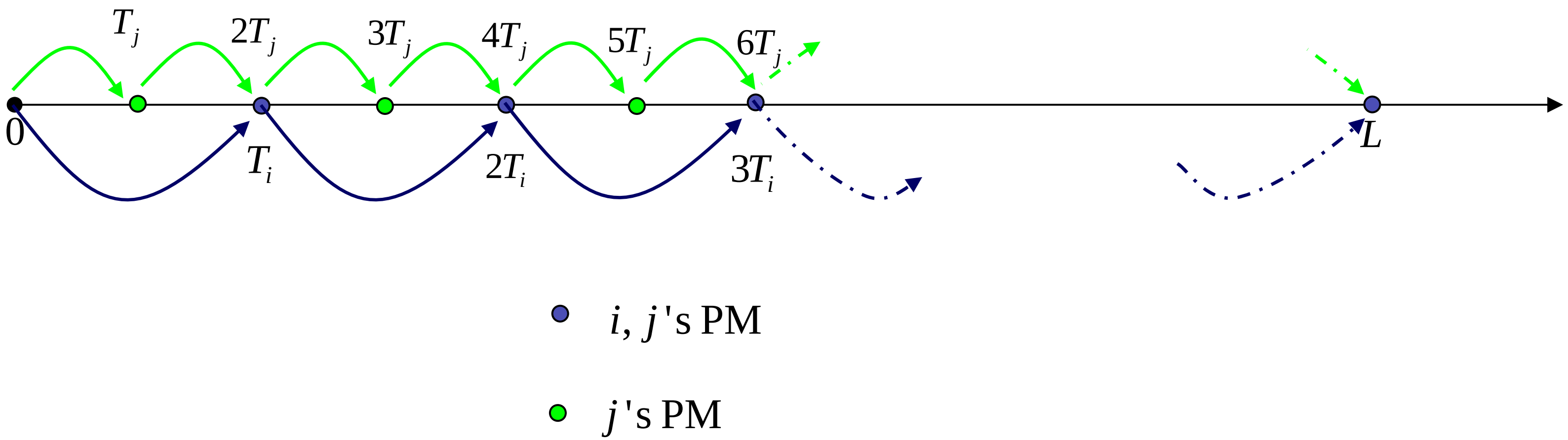
$$T_i = \alpha_{ij} T_j \qquad \forall\ j \in \Omega_i$$

$$0 \le T_i \le L \qquad \forall i \in I,\ \ \alpha_{ij} \in Z^+$$

$$\overline{C}_i = \left(\frac{\left\{ \sum_{m\in F^i} p^i_m (1 - p^{i,m}_{trip})(C^{i,m}_{cm} + \sum_{j\in I\setminus\{i\}} \sum_{m'\in F^j} p^{i,m}_{j,m'} C^{j,m'}_{cm}) \right\} +}{\left\{ \sum_{m\in F^i} p^i_m p^{i,m}_{trip}(C^{i,m}_{trip} + \sum_{j\in I\setminus\{i\}} \sum_{m'\in F^j} p^{i,m}_{j,m'} C^{j,m'}_{trip}) \right\}} \right)$$

Structural Dependency Constraint

$$T_i = \alpha_{ij} T_j \qquad \forall \; j \in \Omega_i$$



j is structurally dependent on i

Proposition[3] - Optimal Solution On $D^{|I|}$

$$D = \left\{ d : d = \frac{L}{n},\ n \in Z^{+} \right\}$$

$$\text{e.g. } L = 10 \ \Rightarrow D = \left\{ \frac{10}{1}, \frac{10}{2}, \frac{10}{3}, \frac{10}{4} \ldots\ldots \right\}$$

$$\mathbf{T}^{*} = (T_1^{*},\ T_2^{*}, \ldots\ldots\ldots, T_{|I|}^{*}) \in D^{|I|}$$

[3] I. Singh, E. Popova and E. Kee, "Optimal preventive maintenance policy under a budget constraint," ICONE16, Orlando, FL, Proceedings Of The 16th International Conference On Nuclear Engineering, May 2008

0-1 Formulation for Preventive Maintenance Model for Multiple Systems

$$x_n^i = \begin{cases} 1 \text{ if } n \text{ PMs are performed on item } i \\ 0 \text{ Otherwise} \end{cases}$$

Maximum number of PMs on i

$$Z_i^+ = \left\{ n \in Z^+ : n \leq \frac{B - \sum_{j \in I \setminus \{i\}} C_{pm}^j}{C_{pm}^i} \right\}$$

$$B \geq \sum_{i \in N} C_{pm}^i$$

0-1 Formulation for Preventive Maintenance Model for Multiple Systems

$$\min_{x_n^i} \sum_{i \in I} \sum_{n \in Z_i^+} \overline{C}_i \, E\left[N^i\left(\frac{L}{n}\right)\right] n \, x_n^i$$

Subject to $$\sum_{i \in I} \sum_{n \in Z_i^+} C_{pm}^i \, n \, x_n^i \le B$$

$$x_n^i \le \sum_{a=1}^{Z_j^+} x_{an}^j \qquad \forall \; j \in \Omega_i \;\; n \in Z_i^+$$

$$\sum_{n \in Z_i^+} x_n^i = 1 \quad \forall \; i \in I$$

$$x_n^i \in \{0,1\}$$

Maximum number of PMs on i

$$Z_i^+ = \left\{ n \in Z^+ : n \le \frac{B - \sum_{j \in I \setminus \{i\}} C_{pm}^j}{C_{pm}^i} \right\}$$

Summary

- PM Optimization for a single system is easy
- PM Optimization for multiple systems is hard
- Efficient algorithm is required