

OA 4202, Homework 0

Nedialko B. Dimitrov

Due Wed. September 29 at the beginning of class

1. Drawing on your past personal experiences, give an example of something that can be modeled as a graph. What are the nodes of the graph? What are the edges? (No more than 3 sentences are necessary)
2. What is the difference between a graph and a multigraph? In an undirected graph with self-loops, are edges described using a tuple, a set, or a multiset?
3. In class we defined several graph-related terms in words and with mathematical symbols. For a graph $G = (V, E)$ and node $a \in V$, define $\text{predecessors}(a)$ and $\text{successors}(a)$ with mathematical symbols.
4. (a) Write the adjacency matrix of the graph in Figure 1, lets name the matrix A .
(b) Write the transpose of A (denoted A^T).
(c) Draw the graph whose adjacency matrix is A^T .
(d) What is the relationship between the graph in Figure 1 and the graph you drew in problem 4c.

5. Write the matrix $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 11 & 0 & 0 & 12 \\ 0 & 13 & 0 & 0 \\ 14 & 0 & 15 & 0 \end{bmatrix}$ in:

- (a) Dictionary of keys (DOK) format
 - (b) Row based linked list (RBLL) format
 - (c) Compressed sparse row (CSR) format
 - (d) Compressed sparse column (CSC) format
6. An adjacency matrix stored in CSR format has the following values:
index pointer: 0,3,4,4,5
columns: 1,2,3,2,2
data: 1,1,1,1,1
Draw the corresponding graph.
7. We are given a matrix A stored in CSR format.
- (a) What is the run time of finding A^T in CSC format? (Hint: For the A in problem 5, try writing A^T in CSC format.)
 - (b) What is the run time for finding A in CSC format?

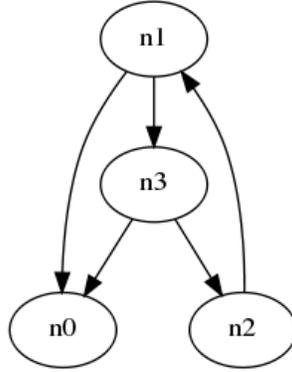


Figure 1: A graph used for a few problems in the homework

8. (a) For an $n \times n$ sparse matrix with m non-zeros stored in CSR format, what is the length of “index pointer,” “columns,” and “data”?
- (b) We have an $n \times n$ sparse matrix with m non-zeros stored as a sparse matrix. We want to change a zero in position (i, j) to a non-zero. This is called the “insert (i, j) ” operation. What is the run time of the “insert (i, j) ” operation if the matrix is stored in DOK format? RBLL format? CSR format?
9. We have an $n \times n$ sparse matrix A with at most k non-zero entries in each row and k non-zero entries in each column. We also have a dense vector v of length n with n non-zeros. In what format should we store A to be able to compute $A \cdot v$ quickly? What is the run time of computing $A \cdot v$? In what format should we store A if we want to compute $v^T \cdot A$ quickly?
10. Give an example of something that can be modeled as a graph, where the resulting graph has over a billion nodes. “Devices on the internet” and “the person-to-person social network of the world” are good examples, but you should try to think of something else. What are the nodes of the graph? How do you know there is over a billion nodes? (any explanation, even Wikipedia is acceptable) What are the edges? What is your guess on how many edges are incident on each node? (any guess is acceptable)