OA 4202, Homework 3.2: Roadblocks in the The Drug Lord's Way

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Due Mon Oct. 18 at the beginning of class

In this homework, we continue our modeling and problem solving to help the Arizona state police capture the escaping drug lord, Tihomir Anastazov.

Background. In the first homework, we modeled Tihomir's movements. We are now working with the state police to disrupt Tihomir's escape as best we can. In this second assignment, we will help compute roadblock locations to guide the Arizona state police.

Assignment. To compute roadblock locations, we have formulated a shortest path interdiction model like the one discussed in class. The inner-problem is modeling Tihomir's escape... a `min` problem for computing the shortest path. In the outer-problem, a `max` problem, we are placing roadblocks to make his shortest path as long as possible. The model all together is a `max min` model. To be able to stick our model into a solver, we have taken the dual of the inner LP, turning the entire model into a `max max` model. The entire model is now a simple MIP that we can solve with GAMS, for example.

Whenever we make a roadblock plan, Tihomir's vast network of informants immediately tells him about it, and he adjusts his escape route. There is just no hiding information from Tihomir. Our model accurately captures the fact that Tihomir knows our every move. Even though he knows the location of our roadblocks, we are still going to do our best to disrupt his escape.

Go to the course webpage and download the archive `3-2-files.zip`. Unzipping the archive will give you access to the following files: `arizona-state.jpg`, `arizona-escape-nodes.pdf`, `arizona_nodes.csv`, `arizona_arcs_set.csv`, `arizona_arcs_data.csv`, `arizona_roadblock_arcs_set.csv`, `spstop.gms`, `cplex.opt`. These files are essentially the same as the ones from the previous assignment, with three exceptions. First, the file `spstop.gms` provides you with the MIP formulation of the roadblock placement problem. You can use that file to complete this assignment. Second, the file `arizona_roadblock_arcs_set.csv` lists the arcs on which we are allowed to place roadblocks. Third, the file `cplex.opt` lists some important MIP solver options for CPLEX.

To get the project files to work for you, you have two options.

1. You can execute GAMS from the command line in the same directory as all the files you downloaded.

2. In the GAMS IDE, you can go to File → Project → New Project, and create a new project with whatever name you like, but in the same directory as all the files you downloaded. Then, in your newly created project, you can open the file `spstop.gms` and run it.

For things to work properly, you have to use CPLEX as your solver. So, don't change the `LP =` and `MIP =` lines at the beginning of `spstop.gms`. Also the CPLEX options file `cplex.opt` must be in the same directory as your `spstop.gms`.

For homework do the following:

1. Read over, and familiarize yourself with the heavily commented GAMS code in `spstop.gms`.

2. Compare the files `arizona_roadblock_arcs_set.csv` and `arizona_arcs_set.csv`. What are the arcs on which we not allowed to place roadblocks? Why aren't we allowing the model to place roadblocks on those arcs?

3. Compute the best roadblock location given 1 roadblock. What is Tihomir's new Escape route, given the single road block?

4. For each value of the number of roadblocks between 0 and 8, compute and record the value of each of: (a) The location of the roadblocks (b) Tihomir's best escape route in response to the roadblock locations (c) The escape distance of Tihomir's best route

5. Looking over the data you've collected, what interesting thing happens with Tihomir's escape plan when we have 3 roadblocks?

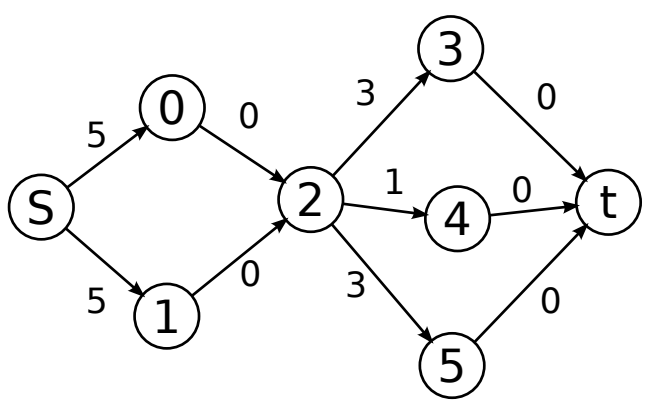
6. Using the data you collected, plot a curve where on the x axis we place the number roadblocks and on the y axis we place Tihomir's escape distance. In general, such a curve is called the *operator resilience curve*. The name comes from thinking of Tihomir as the operator of some "system," in this particular case, his escape route plan. We are attacking that system, in this case, by placing roadblocks. The curve you plotted shows how resilient Tihomir's system is to our attacks, as they grow in number.

7. In the resilience curve you plotted, we see a big jumps in the escape distance as we move from 0 roadblocks to 1 roadblock and as we move from 4 roadblocks to 5. What is happening with Tihomir's escape plans that results in these increase?

8. How do you suppose the resilience curve continues past 8 roadblocks? Why?

9. We say that the solutions to an interdiction problem are *nested* if the attack plan with k attacks is always a superset of the attack plan with $k-1$ attacks. In this problem, this means that as we add the capability for an additional roadblock, we always keep the previous roadblocks we've placed at the same location. Are our roadblock location solutions nested? If yes, briefly describe why they are nested. If no, show a location where the solutions are not nested.

10. Are solutions to interdiction problems nested in general? In the following diagram, suppose we are maximizing the shortest path from s to t. If we are allowed to remove one arc, what is the best arc to remove? What are the two best arcs to remove? (Use visual inspection.)



11. Now, lets change the problem so that a roadblock doesn't simply delay Tihomir by half an hour. We want to make it so that a roadblock effectively makes that arc unusable by Tihomir. To what value should we set the `delay` parameter in the GAMS code? (Hint: The value is computed in the GAMS code.)

12. Plot the new operator resilience curve, under the setting that a roadblock makes an arc unusable. Be sure to record the road block locations and Tihomir's escape plans along the way, because we'll ask a few questions about them later. Hint: Adding a loop that looks like the following one can help save you some time:

```
set numrblks / 0*8 /

LOOP(numrblks,

roadblocks = ord(numrblks)-1;
--- existing code in the file for solving/printing solutions ---
);
```

13. When we have 6 roadblocks, where do we place them? Is that intuitive?

14. For roadblocks numbering between 0 and 5, plot the resilience curves from the first part of the assignment, where a roadblock is a delay of half an hour, and the second part of the assignment, where a roadblock makes an arc unusable, as two lines in the same graph.

    You can think of a roadblock that makes an arc unusable as a "stronger" attack than a roadblock that simply adds a delay of half an hour. Plotting both resilience curves on the same graph, allows us to see how much value we are gaining for having stronger attacks.

15. For the answer in the previous problem, we see that the two curves are the same up to 2 roadblocks and they diverge at 3 roadblocks. What happens at 3 roadblocks to make the curves diverge? Similarly, why are the curves the same up until 3 roadblocks?