Notes 7

Multi-commodity flows.

- suppose we have a graph G,
  - supplies of rice and flower / demands
  - capacities on the arcs, costs for transport across arcs.

- Can we use regular min-cost flow to clear the market? Why not?

  - have to make sure a "rice" demand gets satisfied with a "rice" supply.
    ... have to keep track of multiple commodities.

as an LP: K - set of commodities

- y_{ij}^k : flow of commodity k on arc (i,j).
- $b_{(a)}^k$: supply/demand of commodity k on node a.
- c_{ij}^k : cost of commodity k on arc (i,j)
- u_{ij} : capacity of arc (i,j)

$$\min_{y} \sum_{k} \sum_{(i,j)} c_{ij}^k y_{ij}^k$$

s.t.

$$\sum_{(i,a)} y_{ia}^k - \sum_{(a,i)} y_{ai}^k = b_{(a)}^k \qquad \forall a \in V, k \in K$$

$$0 \le y_{ij}^k \qquad \forall (i,j) \in E, k \in K$$

$$\sum_{k} y_{ij}^k \le u_{ij} \qquad \forall (i,j) \in E, k \in K$$

capacities for each commodity → $c_{ij}^k \le y_{ij}^k \le u_{ij}^k$

How are multi-commodity flows relevant to many real-world modeling problems?

- e.g. we have a city with neighborhoods A, B, C, D, ...

we have morning traffic demands:

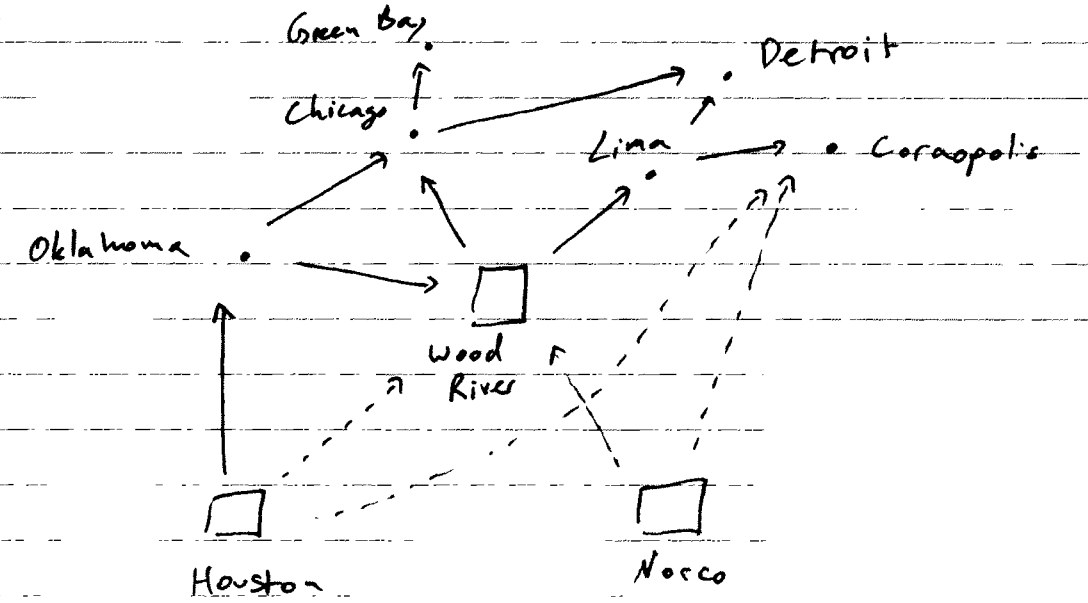
2000 from A to C
4000 from D to B
7000 from C to A.

can the city's transportation network handle the traffic?

Shell Oil Company (10-51 Rardin)

- Shell supplies three products to the midwest
  - gasoline, jet fuel, fuel oil

- Shell's supply network looks like this:



- solid lines are pipelines. dashed lines are barges.

- squares are refineries

- let d_{ip} be the demand for product p on node i.

- let b_{ip} be the production capacity for product p on refinery i.

- barge capacity is unlimited, but a pipeline from i to j can carry at most u_{ij} total product.

- we have cost r_{ip} for producing product p in refinery i

- we have cost c_{ij} for shipping any product from i to j.

- Formulate as multi-com flow.

- Profit for selling product?