# OA 4202, Homework 3.3:
# A More Realistic Drug Lord

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Due Tue Oct 25 at the beginning of class

In this homework, we will attempt to add some more realism in our attempts to help the Arizona state police capture the escaping drug lord, Tihomir Anastazov.

**Background.** In the first homework, we modeled Tihomir's movements. In the second homework, we used a MIP to compute roadblock locations and did some analysis of Tihomir's operator resilience curve. In this homework, we change the model to make it more realistic.

**Assignment.** For this homework assignment, the starting points are the same files as the previous homework.

Go to the course webpage and download the archive `3-2-files.zip`. The archive contains the same files it did in Homework 3.2. I've added the files to support the COINCBC solver. For things to work properly, you should use either CPLEX or COINCBC as your solver. The options file for the solver you are using, either `cplex.opt` or `coincbc.opt`, must be in the same directory as your `spstop.gms`.

For homework do the following:

1. So far in this assignment, we have been modeling Tihomir as seeking the shortest escape distance from Flagstaff. In reality, he is likely to seek the path that maximizes his probability of escaping arrest. For brevity, lets call the probability that Tihomir escapes arrest his *probability of evasion*. Let's switch to modeling that probability of evasion instead.

   Assume the following things:

   - There are 3.4 abstract arc units per mile
   - Traffic moves at a constant 65 miles per hour
   - For every hour on the road, there is a 1% chance of being pulled over randomly.

   We'll use a continuous model to translate from hours on the road to probabilities of evasion. In other words, the model looks something like this Prob_evasion $= e^{-r \cdot \text{hours}}$, where $r$ is a "rate of capture" that you can calculate using the above assumptions. When we plug in 0 hours in the above formula, we should be getting a probability of evasion of 1. When we plug in 1 hour into the formula, we should be getting a probability of evasion of .99.

   Let's denote the probability of evasion on arc $(i, j)$ with the symbol $p_{ij}$.

   Open `arizona_arcs_data.csv` and add a column to the right of the "Distance" column, with the header "POE" for probability of evasion. For each arc in the network, use the above assumptions and the continuous model to compute Tihomir's probability of evasion if he traverses that arc alone. The values you computed are the $p_{ij}$'s. Store them in the data column you created. Since we are working with numbers very close to 1, be sure to use at least 10 decimal places of precision.

So that I can spot-check your work, give the new data column's values for the arcs ("Wickenburg","Aguila") and ("Parker", "IFortyWest").

2. Tihomir assumes that the probability of getting pulled over in any two stretches of time on the road is independent. So, he is trying to find the path that maximizes his total evasion probability: $\max_{\text{escape paths } P} \prod_{(i,j)\in P} p_{ij}$.

   Use another data column in `arizona_arcs_data.csv` and the $-\log$ transformation we discussed in class to turn Tihomir's "maximum evasion probability path" problem into a shortest path problem. Give the new data column the name "MINLOG."

   So that I can spot-check your work, give the new data column's values for the arcs ("Wickenburg","Aguila") and ("Parker", "IFortyWest").

3. Using your new "MINLOG" data column, and the code in `spstop.gms` as a starting point, compute Tihomir's maximum evasion probability path out of Flagstaff.

   As long as your newly saved `arizona_arcs_data.csv` file does not have any spurious quotations or spaces in it, your new data columns will be read into GAMS. You can refer to the data in your new column as `arcdata(i,j,'MINLOG')`.

   What is Tihomir's maximum evasion probability path? What is his probability of evasion on this path? (Hint: In all of the GAMS lines that look like `put 'transit cost=',Z.l:10:1 / ;`, the last number after the colon, the 1, specifies the decimal places of percision. Try setting this to 7.)

4. Assume that if Tihomir runs a roadblock he will get detected with 100% probability. Compute the best roadblock locations for a number of roadblocks ranging between 0 and 5. Be sure to record the locations of the roadblocks and Tihomir's best response evasion route.

   Plot Tihomir's operator resilience curve, where on the horizontal axis we have number of roadblocks and on the vertical axis we have his evasion probability. (Hint: Tinker with the `put 'transit cost=',Z.l:10:1 / ;` line once again, so that GAMS automatically puts out evasion probabilities instead of unintelligible $-\log$ costs.)

5. Look at Tihomir's escape routes and the computed roadblock locations. Is anything different from problem 12 in Homework 3.2?

6. If we were to change the assumption that "for every hour on the road, there is a 1% chance of being pulled over randomly." To some other non-zero percent chance of being pulled over, would that change the answers significantly? Why?

7. Lets increase the realism slightly more by assuming that if Tihomir escapes to another US State, then there is a 75% chance that he will be caught in the future. While, if he escapes to Mexico, he can roam freely for his remaining days.

   Look at the "POE" column in `arizona_arcs_data.csv`. To add in our new assumption about escaping to Mexico vs. another US State, you should tinker with the arcs that currently have an evasion probability of 1.

   What is Tihomir's maximum evasion probability route, when there are no roadblocks, under our new model? What is his evasion probability?

8. Compute the best roadblock locations under the new model for roadblocks ranging from 0 to 6. What do you notice about Tihomir's escape routes? What is his escape route when we have 5 roadblocks in place?

9. Ok, in the previous problem we were still able to cut off all of Tihomir's escape routes with 6 roadblocks. But, that may not be too realistic. For example, let's suppose that Tihomir has had some lead-time on us, so we know that putting a roadblock on any arc leaving out of Flagstaff will be useless.

   Create a new model in which we are not allowed to place any roadblocks on arcs leaving Flagstaff. Do this *without* touching the `arizona_roadblock_arcs_set.csv` file, simply by adding a new constraint to the `BlockPlacementMIP` model in the GAMS file.

   What is the GAMS code of the new constraint you added?

10. Solve our new model for number of roadblocks ranging between 0 and 20. (If you have access to both CPLEX and COINCBC, try doing it once in each of these solvers. Notice anything? If you don't have access to both, skip this part.) How many roadblocks do we need before we cut off Tihomir's escape completely? What interesting thing happens as we move from 5 roadblocks to 6 roadblocks?

11. Using the picture in `arizona-escape-nodes.pdf` and whatever drawing program you know, plot the road block locations for 12 roadblocks and 13 roadblocks. Visualizing the solutions like this is a key part of understanding what the model is doing, using the results in real life, and presenting results to others.

12. Now lets go back to pretending that Tihomir doesn't have a head start on us. So, we are once again able to put roadblocks on the arcs going out of Flagstaff.

    However, lets introduce one final complication in our roadblock deployment plans. Suppose that each city has a single police department, with a limited number of police officers, and so the city cannot put up too many road blocks. In fact, the best we can do, is instruct the police department of each city to send out their police cars on at most one road leaving the city and put up a roadblock there. We'll pretend that the state highway patrol can cover for the "cities" that are just highway intersections. We still have a total number of roadblocks limit.

    Model this new complication by tinkering with the code in the GAMS file. This should only take you around 4 or 5 lines of GAMS code.

    What did you do in the GAMS file?

13. How many road blocks does it take before we can stop Tihomir's escape to Mexico? If we remove the roadblock limit entirely, but keep the limit that each city can block only one of the roads going out of the city, can we stop Tihomir?

14. Take any one thing you don't like about our model, and model it in a better way. Solve your new resulting model. Explain what you did. Did you get significant differences in the answer?

    This is an open-ended question, so do something that you think is interesting and a good learning experience for you.