Red Team Modeling Project

Executive Summary

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Background. Johnny Joe's is an emerging potato chip conglomerate. We have potato chip manufacturing/distribution plants in several cities throughout the United States. Annual demands vary among these plants. We also have numerous suppliers of raw potatoes located across the United States and Canada. One of our largest overhead costs is the cost of shipping the raw potatoes from our producers to our manufacturing/distribution plants. Our goal is to minimize these costs. Realizing the potential of being overtaken in the worldwide potato chip market, Doritos is rumored to be considering aggressive options to sabotage our continued growth. They have two methods of attack. In case 1, Doritos drives up the prices of potatoes from key suppliers through aggressive bidding. In case 2, Doritos buys up all the available potatoes from certain key suppliers, effectively eliminating that supplier from our options. Our analysis first looks at which suppliers to choose and what quantity to order from each supplier in the absence of an attacker. Then we focus on identifying the most vulnerable points in our potato shipping model in the presence of an attacker.

Mathematical Model. We model the real-world problem using a modified Min-Cost Flow model. The single real-number-valued measure of effectiveness is the total cost. Costs are incurred through shipping costs and shortage costs. The demand that we are trying to clear drives the flow in our model. So, in other words, our goal is to clear the demand at minimum cost. The mathematical model is shown below.

Notation: i: nodes (alias j, a)
c_{ij} = shipping cost in \$ per cwt(centum weight)-mile between nodes i and j
d_{ij} = delay cost in \$ per cwt(centum weight)-mile between nodes i and j
X_{ij} = (binary variable) attack edge (i,j) or not
Y_{ij} = flow between nodes i and j
s_j = shortage cost per cwt of potatoes at node j
UD_j = unsatisfied demand at node j in cwt of potatoes

Objective: $$\min \sum_{(i,j)\in E} (c_{ij} + d_{ij} \cdot X_{ij}) Y_{ij} + \sum_{j|b(j)>0} s_j UD_j$$

s.t. $$\sum_{(i,j)\in E} Y_{ia} - \sum_{(i,j)\in E} Y_{ai} = b(a) \qquad \text{(NETFLOW)}$$

$$Y_{ij} \le u_{ij} \qquad \text{(CAPACITY)}$$

$$Y_{ij} \ge 0 \qquad \text{(NON-NEGATIVE FLOWS, LOWER BOUND)}$$

As shown in the mathematical above, we modify the standard Min-Cost Flow model in a couple of ways. First, we add a cost term in the objective function to account for unsatisfied demand. Each unit of unsatisfied demand is multiplied by the shortage cost at the associated demand location. Shortage costs vary among demand locations. For example, the shortage cost is higher for New York than for St.

Louis. In other words, a shortage in demand at our New York plant results in a greater loss of revenue than at our St. Louis plant. The second modification we made (not shown above) is to split each supply node into two separate nodes. This allows us to more realistically model our attacker. Out attacker does not pick individual routes to attack or drive up the cost. Rather, our attacker chooses to attack certain suppliers thereby influencing all routes emanating from that supplier. In real-world English, our competitor chooses suppliers and (case 1) aggressively bids for the potatoes from those suppliers thereby driving up the cost for all shipments coming from those suppliers or (case 2) buys out all available potatoes from that supplier.

Analysis. Running our baseline model with no interdictions results in a total cost of just over $3 million. All demands are met. We use the suppliers located at Bakersfield, Bangor, Chippewa Falls, and Colorado Springs. To implement case 1, we set the delay parameter in the mathematical model to $40 per cwt-mile. This value corresponds to roughly 50% of the maximum shipping cost per cwt-mile. The operator's resilience curve for case 1 is shown below (figure 1). Not surprisingly, our first interdiction attacks the supplier located in Maine. This supplier is located relatively close, geographically, to several of our large demand facilities, namely New York, Philadelphia, and Boston. Going from zero to one and one to two interdictions results in a jump in our total cost of around $2.5 million. All subsequent increases in the number of interdictions only add about $1 million to our total cost each time.

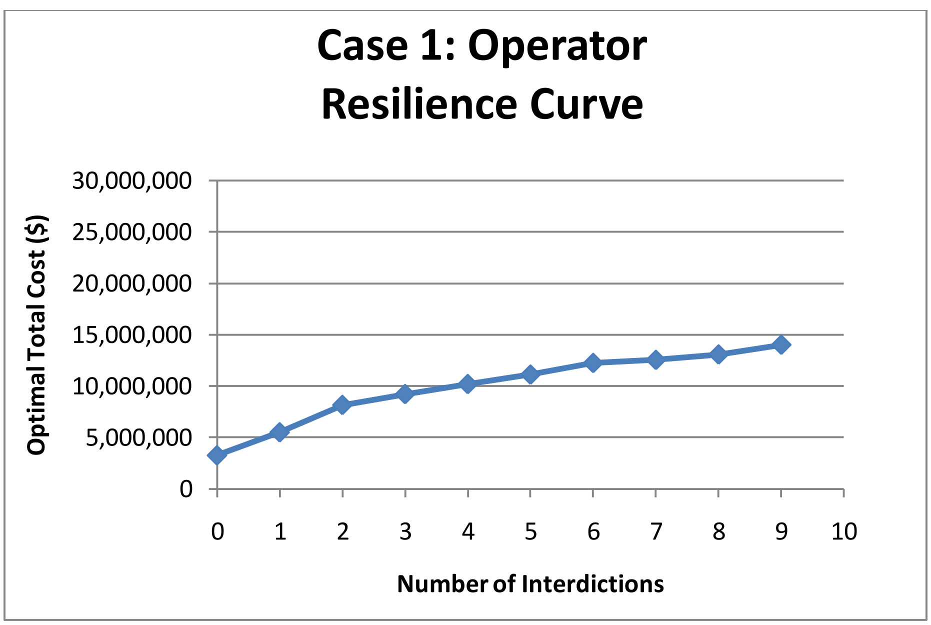

Figure 1 - Operator resilience curve for case 1.

To implement case 2, we set the delay parameter in the mathematical model to be very large to effectively eliminate the chosen supplier. The operator's resilience curve for case 2 is shown below (figure 2). In this case, with just one interdiction (one supplier completely bought out), our total cost jumps from around $3 million to around $5.5 million. With each additional interdiction, our total cost increases similarly. Interestingly, the attacker does not always choose the same suppliers to attack as in case 1. Whereas in case 1 the attacker seemed to favor those suppliers located closest to our demand facilities, here in case 2 the attacker favors the largest supply facilities. Up to seven interdictions, we are still able to satisfy demand at each location in case 2. However, with eight interdictions, we have shortages at six of our ten plant locations. As a result, we pay the corresponding shortage cost for each unit of shortage at each location. This explains the approximate $3 million jump in total cost going from seven to eight interdictions in the graph below. The jump in total cost going from eight to nine interdictions is even larger at around $10.5 million.

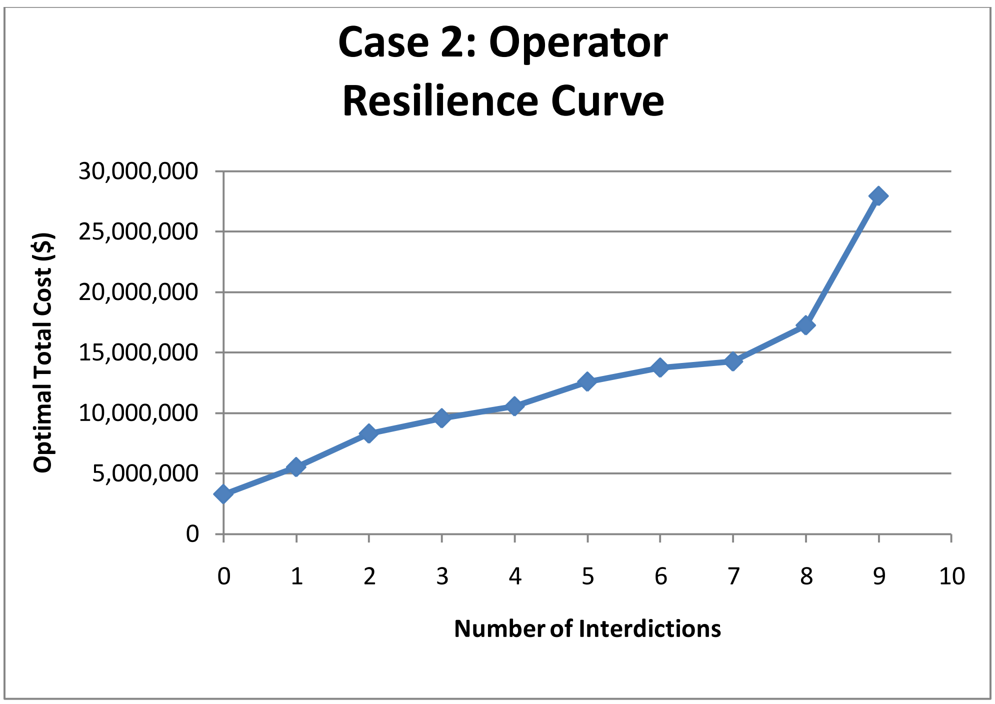

Figure 2 - Operator resilience curve for case 2.

Summary and Conclusion. Based on the analysis, it is clear to our company that we need to foster the relationships with four key suppliers: Bangor, MA; Chippewa Falls, WI; Bakersfield, CA; and Billings, MT. The first two are located in relatively close geographic proximity to our largest demand facilities and thus offer great value in terms of shipping costs. The latter two have extremely large available supplies of potatoes that could conceivably meet our nationwide demands single-handedly. Preventing or reducing attacks in our relationships with these four suppliers makes us resilient to either of the attack cases of our competitors considered above.