OA4202 - Network Flows and Graphs
Executive Summary: London Subway System.

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Abstract— **The primary question we sought to answer is, does the *London Underground* have the resources to support the *2012 London Summer Olympics*. In the process of seeking this answer we determined that there were several other network problems that needed to be addressed.**

1) **On which paths are the visitors most likely to travel from the hotel areas to the *Olympic Park*.**
2) **What's the maximum number of visitors the *London Underground* is able to transport to the *Olympic Park* at the same time.**
3) **Which fraction of this capacity is actually in the vicinity of the *Shortest Path*.**

The *Executive Summary* is as follows:

I. INTRODUCTION TO THE PROBLEM

Current figures published by the London Olympic personal show 3.5 Million tickets sold to 850,000 people. This equates to an average of about four tickets per person. Expected ticket sales are to reach 10.8 Million with 2.8 Million of those associated with venues outside of London. Focusing on the London venues only, there will be approximately 8 Million tickets held by above 2 Million people in London area during the 16 days of the Summer Olympics.

The majority of the hotels are located within five miles of the center of London additionally there is a numerous amount close to Heathrow airport. There are 21 Olympic venues throughout the London area with Olympic Park hosting six of those venues as well as the athlete's Olympic Village. Out of the estimated 2 Million visitors we calculate in excess of 900,000 people whose destination will be Olympic Park. Olympic Park venues range from 6 miles to 8.7 miles from city center.

Based on these factors we concluded the logical end point for our scenarios is Olympic Park . To choose the capacity of our scenarios we selected the single sport venue with the largest capacity which is Olympic Stadium (80,000), which is also the location of the opening and closing ceremonies.

A. Assumption

In order to reduce the complexity of the model, we made the following assumptions:

- The capacity of a line stays constant from start to finish.
- To keep the model from unnecessarily changing lines we implemented transfer costs (penalty) of 30 minutes to change.
- We assume that the entire period of the Olympics is considered as *Peak*. Therefore we base our calculation on the capacity under *peak conditions*.
- All calculations are based on current conditions of the network.
- We do not include the reduced capacity because of the daily usage of the tube. This means, that we assume that the full capacity of the network can be used to support the Olympics.

B. Mathematical Model and Critical calculations

We used the standard GAMs model for Shortest Path/ Interdiction, Maximum Flow/ Interdiction and Minimum Costs Flow. To support the different scenarios an editor was generated to replace all hard coded parameters (typically the RHS of the constraints) by parameters GAMs could read-in at runtime. Following a short example:

```
SCALARS numberStartPoints /
$INCLUDE  data/numberStartPoints.csv
/;
```

- Due to lack of actual distance between stations data, several critical calculations were required to ensure the model accurately reflected real time data.
  - To calculate a distance between two stations, the X,Y coordinates on the map were compared to the actual distance between two points to obtain a conversion factor. Using this factor allowed to transfer the distances from the artificial map units into miles.
  - The time between stations is calculated based on the estimated distance in miles and the average speed of a train (23 miles/hour).
- The maximum number of trains on a line is based on the total number of available trains during peak hours and the amount of peak hours.
- The capacity of a line is based on the maximum capacity over all train types used on a line and the maximum number of trains of that line.
- The number of passengers to transport is calculated based on the number of events in one time block and the venue capacity (tickets are sold in time blocks vice by events).

C. Validation

One way to validate our model is to compare the results we get as a *Shortest Path* for a given start- and end point with the results returned by the *London Journey Planner*. The current model matches the results for all combinations we tried.

D. Implementation

To implement our model we used the following environments:

- GAMs, to determine the *Shortest Path*, the *Maximum Flow* and the *Minimum Cost Flow*.
- MySQL, to hold all data associated with the *London Underground* (Entity-Relationship Diagram attached).
- Java, to develop a *Scenario Editor*, export the data to GAMs and present the results graphically.
- R, to generate *Operator Resilience Curves*.

E. Network

Figure 1 shows the complete network of the *London Underground*. Corresponding to this real network we built our model to match figure 2.

Fig. 1. Actual London Underground Tube Map.

Our network consists of following data:

- The nodes are the substations of the London underground. A physical station can have several substations which represent the connections to different lines.
- The arcs are the line segments between the substations. We also added artificial arcs to connect all substations of the same station.
- The properties associated with each arc are distance, costs and maximum capacity. For all real arcs this data was given or estimated. Data for the artificial arcs is zero distance, 30 minutes delay and a infinite capacity.
- Start and End nodes. In order to support more than one start and end point for travel, we added one *super source* and one *super sink* node. For maximum flow and minimum costs scenarios these nodes are the only ones with demand and supply.

II. Leading Problem

The primary question to answer is: "Is the the *London Underground* able to transport the additional number of passengers ?" To answer this questions we needed to find the path on which the visitors are most likely to travel from the hotel areas to the *Olympic Park*. After determining these routes we could calculate the capacity along them or in the vicinity of them.

A. Shortest Path

The shortest path problem allowed us to determine the relevant nodes and arcs on the routes from the hotel areas to the *Olympic Park* (figure 3). From these results we discovered

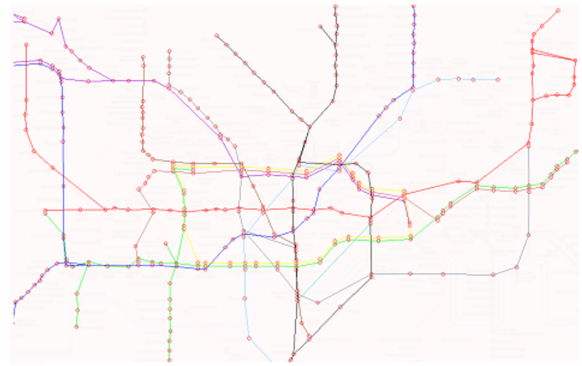

Fig. 2. Abstract representation of the underground tube map.

vice the actual station that nodes needed to be substations inside the stations. By creating the substations we could add a penalty for changing lines. As the *Operator Resilience curve*

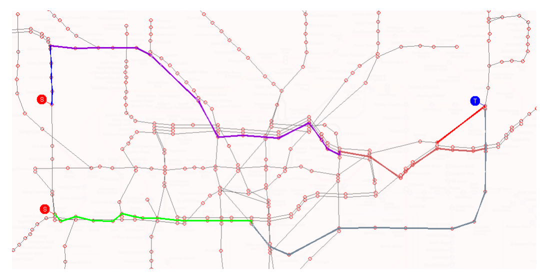

Fig. 3. Shortest Path from "Acton Town" and "Alperton"(both are in the vicinity to hotels) to Stratford (closest station the Olympic Park).

(figure 4) shows, the network in the center of London is well connected, and can compensate one interdiction with nearly no increase in travel time. Two ore more interdictions made it necessary to change lines which incues the penalty time for changing lines.

B. Maximum Flow

We primarily use the maximum flow to get the upper bounds for the minimum costs flow. This capacity includes the capacity of routes which are to far from the shortest path and will probably be considered unacceptable by most visitors.

Maximum Flow		
# Interdictions	Capacity of flow	% of max
0	114,264	100
1	79,892	70
2	47,704	42
3	23,192	20
4	0	0

Figure 5 shows the similarity between the interdictions in shortest path and max flow problems, under identical conditions.

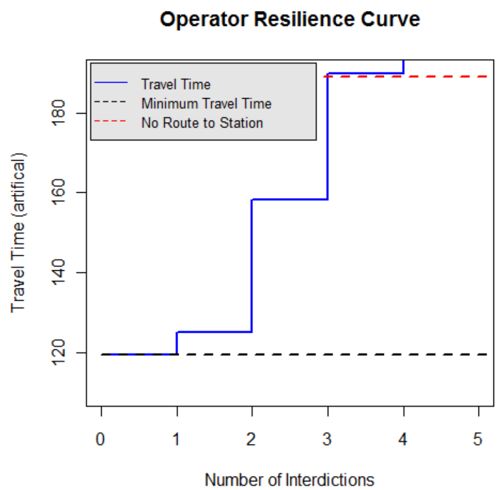

Fig. 4. Increasing travel time for the line segment from “Acton Town” to “Mile End” (line segment is located on route to Olympic Park)as a result of increasing interdictions.

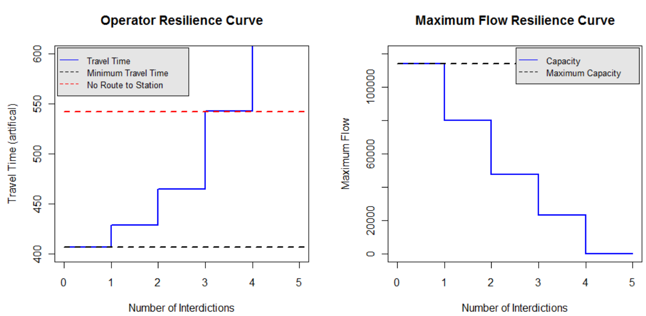

Fig. 5. The image on left shows the increasing travel times (sum) for three line segments from “Acton Town”, “Alperton” and “Amersham” to the Olympic Park as a function of the increasing number of interdictions. Image on the right shows the decreasing capacity for the same starting and ending points.

C. *Minimum Cost Flow*

The sequence of figures 6, 7 and 8 reflect screen shots of the used and unused capacity in the proximity of the shortest path while increasing the number of passengers per hour.

III. Results

We conclude the following results:

- Passengers should be encouraged to take alternate routes avoiding the center of London. The *London Overground (DLR)* connects all tube lines and can disburden the tube system. This will also reduce the increased amount of time it takes while the network is facing interdictions (figures 3, 4). Therefore reducing the ticket price for the DLR can help to avoid overcrowding of the lines close to the *Olympic Stadium*.
- Theoretically the capacity of the complete network is sufficient. However due the lack of redundancy in the lines heading towards *Olympic Park*, more than 2 interdictions will reduce the capacity by about 58 percent (figure 5).
- The capacity along or in the vicinity of the Shortest Path is shown in the figures 6, 7 and 8. The figures reflect that the shortest path support up to 10,000 passengers and the paths in the vicinity of the shortest path support up to 80,000 passengers. Depending on the amount of time the people are willing to spend on the train the routes supporting up to 120,000 people may be unacceptable.

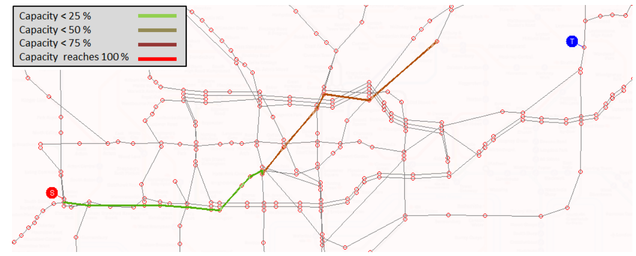

Fig. 6. Minimum Cost Flow pushing 10,000 people from “Acton Town” to the Olympic Park.

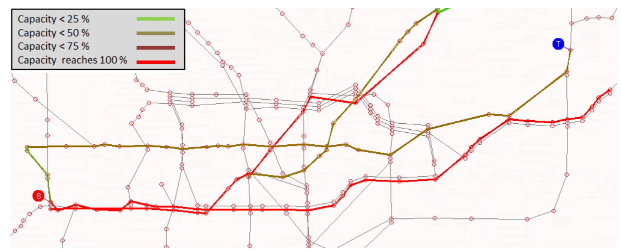

Fig. 7. Minimum Cost Flow pushing 80,000 people from “Acton Town” to the Olympic Park.

IV. Future Research

The biggest improvement to the results can be achieved by

- Incorporating other forms of transportation to the network. Specially including the *London Overground (DLR)* with the connections to the tube would result in more realistic predictions.
- Obtain accurate data to replace the estimated distance between stations by the actual distances of the tracks.
- The estimated time it takes between stations can be replaced by the time given by the train schedules.

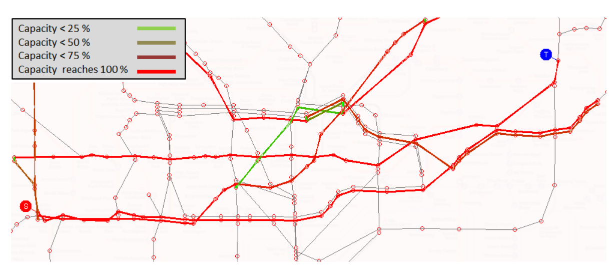

Fig. 8. Minimum Cost Flow pushing 120,000 people from “Acton Town” to the Olympic Park.