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Israel Traffic Model: Jerusalem to Tel-Aviv

Background scenario.

We set out to study a real world traffic network that Yuval Nevo and many others encounter on a daily basis - the Jerusalem to Tel Aviv commuter network. During rush hour, traffic is a serious problem and frequent accidents compound the problem. Based on their own knowledge of the network and the provided traffic reports, drivers have the dilemma of which course to choose.

Methodology.

Abstracting the real-world traffic network in the area, we developed a manageable network for this study that encompasses the primary commuter routes used. The edges in our graph are the various main roads between Jerusalem and Tel Aviv. The nodes are the primary road intersections and Jerusalem and Tel Aviv. Capacity parameters are determined according to our personal knowledge and information about the number of lanes and the permitted speed in each route. We gathered the traffic conditions based on radio traffic reports, and determined the value of the supply, demand accordingly, to develop realistic morning commuter traffic conditions.

We make several general assumptions regarding the structure of the network:

- The model is static, i.e. the traffic has reached equilibrium and the supply and demand do not change.
- To avoid non-linearity, we assume discrete traffic conditions.
- Possible routes only take into account highways, which results in a coarse network.
- All traffic is coming from the East, North, South, and Jerusalem and it all flows to Tel Aviv. Therefore, other cities in the area that could be sources of traffic and traffic flowing the other direction are ignored.
- Lastly, we assume that an accident or other obstacle creates a fixed delay, which depends on the level of traffic congestion but not on the length of the road.

Model Formulation.

The overall measure of effectiveness (MOE) is the time it takes to drive from Jerusalem to Tel Aviv. Therefore, we need to estimate the average velocity on each road, but we also wanted to base this on road conditions. We used an indexing construct to accomplish this distinction. Edge flow variables, costs, and delays are all indexed by the set $k = \{1, 2, 3\}$. Essentially, each edge in the network is modeled as three edges based on the index k. We can therefore associate each value with the average velocity of cars in the road to account for congestion levels.

An index of 1 corresponds to congestion up to 20 cars per minute and an average velocity of 65 km/h, 2 to congestion of 20-40 cars per minute and an average velocity of 35 km/h, and 3 to congestion of 40-60 cars per minute and an average velocity of 10 km/h. Sixty cars per minute is the maximum capacity, but we also account for the number of lanes in a road, so an edge upper bound is equal to 20 * the number of lanes. Similarly, the delay from an accident is indexed by the level of congestion. An index of 1 equals a delay of

10 minutes, 2 equals a delay of 30 minutes, and 3 equals a delay of 60 minutes. We used the minimum cost flow model shown below to generate the traffic conditions. The objective in this model is to minimize the overall amount of time that the cars have to spend on the roads.

$$z = \sum_{i,j,k} y_{i,j,k}\left[c_{i,j,k} + d_k x_{i,j}\right]$$
$$s.t.$$
$$\sum_{i,j,k} y_{i,j,k} - \sum_{i,j,k} y_{j,i,k} = b_j$$
$$y_{i,j,k} \le u_{i,j,k}$$

We then used a shortest path model to find how much time it takes to get from Jerusalem to Tel Aviv, in which the costs are determined according to the traffic conditions, i.e. based on the solution of the minimum cost flow problem. To determine which road demonstrators should block to maximize the total delays, we used the dual of the minimum cost flow model.

$$z = \sum_{i,j} A_{i,j} \cdot \overline{C}_{\bar{y}_{ij1}, \bar{y}_{ij2}, \bar{y}_{ij3}}$$
$$s.t.$$
$$\sum_{i,j} A_{i,j} - \sum_{i,j} A_{j,i} = \begin{Bmatrix} -1 \\ 0 \\ 1 \end{Bmatrix}$$

Analysis results.

The initial analysis of the overall network resilience considering up to six roadblocks is depicted in Figure 1 below. The orange line represents the true operator resilience curve for the model in that it accounts for the total traffic flow in the model and is monotonic. It was interesting to note that the purple line, representing a single driver from Jerusalem to Tel Aviv and is scaled by a factor of 100 for comparison purposes, is non-monotonic. This resulted from the transfer of congestion to alternate paths that actually allowed a single driver to then decrease their travel time with the selection of an optimal path.

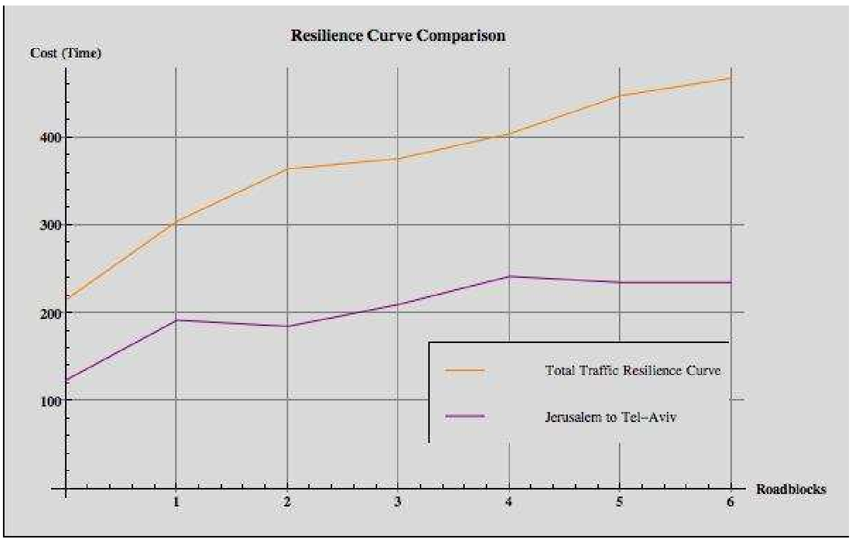

Figure 1 - Resilience Curve Comparison

Now, in most attacker-defender models, the attacker intentionally tries to damage the defender as much as possible. However, traffic accidents occur randomly without intent to disrupt traffic. We addressed the issue of planning an optimal route under stochastic conditions by assuming there is a certain probability of an accident between Jerusalem and Tel Aviv and that accidents are equally likely to occur anywhere along the network. The probability of an accident on a specific edge is a ratio of the length of a road to the total length of all roads. Also, we assume that there can be no more than one accident and that the driver is facing several distinct alternatives. To find the different alternatives, we ran the model repeatedly, each time with an accident in a different location, and checked the shortest route in that scenario. This produced three distinct alternatives which, from Yuval Nevo's personal experience, are realistic.

Figure 2 - Alternative Comparison Under Uncertainty.

For each alternative, we checked how much time a driver would need to get from Jerusalem to Tel Aviv for all possible scenarios. Then, we calculated four indices: the expected time (an expectation over the different scenarios), the worst case scenario, the expected regret (the different between the optimal route given the scenario and the alternative given the scenario), and the worst regret. When comparing the three alternatives, we saw that Alternative 1, the shortest path given no accidents, is in fact the worst. The other two alternatives are harder to compare, as the comparison depends on the criteria used.

Summary.

With a fairly simple network model it was possible to generate very realistic results. The overall traffic flow that was generated as well as the alternative paths chosen all reflect actual conditions quite closely. In terms of robustness, we were able to answer many questions of interest with the same model construction. For example, with minor changes to source node designation we could consider optimal locations for commuter to live in the area. Also, our model is easily extendable to other traffic networks with the necessary data files. Although the network we studied for this project was quite small, the model is scalable to much larger networks.

Lastly, the ability to handle uncertainty greatly increased the realism and applicability of our results. Again, the model was able to reflect reality very closely in this aspect and provide the user with a valuable decision support tool.