OA 4202, Homework 0

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1. (a) Please tell me about the previous job you had, before coming to NPS. Where were you, what were some typical problems you had to solve? No long description is necessary, but this question really helps me know more about you, and sometimes helps me create relevant examples later in the class.

   (b) Describe one part of that past personal experience that you think may be modeled using a graph.

2. What is the difference between a graph and a multigraph? In an undirected graph with self-loops, are edges described using a tuple, a set, or a multiset?

3. In class we defined several graph-related terms in words and with mathematical symbols. For a graph $G = (V, E)$ and node $a \in V$, define predecessors(a) and successors(a) with mathematical symbols.

4. (a) Write the adjacency matrix of the graph in Figure 1, lets name the matrix A.

   (b) Write the transpose of A (denoted A^T).

   (c) Draw the graph whose adjacency matrix is A^T.

   (d) What is the relationship between the graph in Figure 1 and the graph you drew in problem 4c?

5. Write the matrix $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 11 & 0 & 0 & 12 \\ 0 & 13 & 0 & 0 \\ 14 & 0 & 15 & 0 \end{bmatrix}$ in:

   (a) Dictionary of keys (DOK) format

   (b) Row based linked list (RBLL) format

   (c) Compressed sparse row (CSR) format

   (d) Compressed sparse column (CSC) format

6. An adjacency matrix stored in CSR format has the following values:
   index pointer: 0,3,4,4,5
   columns: 1,2,3,2,2
   data: 1,1,1,1,1
   Draw the corresponding graph.

7. We are given a matrix A stored in CSR format.

   (a) What is the run time (in big-Oh) of finding A^T in CSC format? (Hint: For the A in problem 5, try writing A^T in CSC format.)
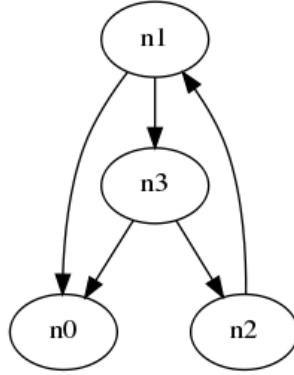
Figure 1: A graph used for a few problems in the homework

(b) What is the run time (in big-Oh) for finding A in CSC format?

8. (a) For an $n \times n$ sparse matrix with m non-zeros stored in CSR format, what is the length of "index pointer," "columns," and "data"?

   (b) We have an $n \times n$ sparse matrix with m non-zeros stored as a sparse matrix. We want to change a zero in position (i, j) to a non-zero. This is called the "insert (i, j)" operation. What is the run time (in big-Oh) of the "insert (i, j)" operation if the matrix is stored in DOK format? RBLL format? CSR format?

9. We have an $n \times n$ sparse matrix A with k non-zero entries in each row. We also have a dense vector v of length n with n non-zeros. What is the run time (run time is always in big-Oh) of computing $A \cdot v$ if A is stored in any of our sparse matrix formats? Write pseudo-code for computing $A \cdot v$ if A is stored in DOK format. (Hint: You may iterate over the hash table, receiving the key (i, j) and the data value d in each iteration. However, it is impossible to control the order in which the iteration is done.)

10. Give an example of something that can be modeled as a graph, where the resulting graph has over a billion nodes. "Devices on the internet" and "the person-to-person social network of the world" are good examples, but you should try to think of something else. What are the nodes of the graph? How do you know there is over a billion nodes? (any explanation, even Wikipedia is accep5tible) What are the edges? What is your guess on how many edges are incident on each node? (any guess is acceptable)

    Can you give a graph whose full (dense, as in zeros included) adjacency matrix has a billion entries?