OA 4202, Homework 2

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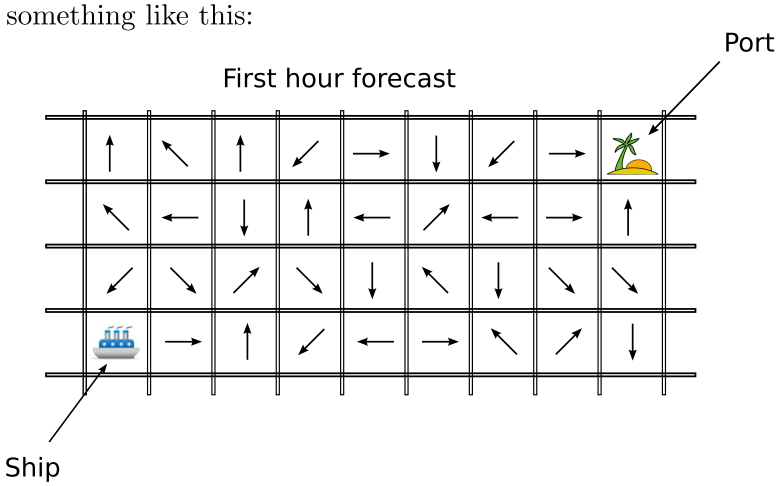
1. Run Dijkstra's algorithm on the graph in Figure 1 with start node 0. Specifically,
   (a) List the order in which nodes are popped off the priority queue.
   (b) For each node u, list the sequence of values given to $\tilde{d}(s, u)$.
   (c) Draw the final search tree of the algorithm.

2. Give a small example of a graph with one negative length edge where there are no negative length cycles but Dijkstra's algorithm doesn't work. Give a short explanation about why the algorithm fails to work in your example.

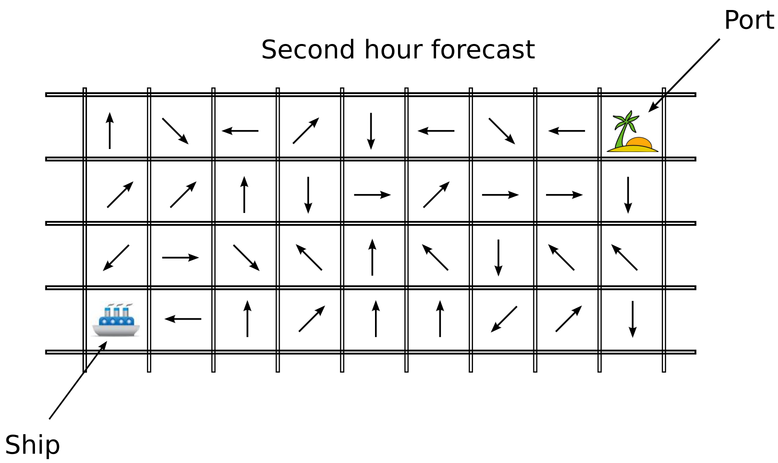
3. (AMO)
   5.21. Professor May B. Wright suggests the following method for solving the shortest path problem with arbitrary arc lengths. Let $c_{\min} = \min\{c_{ij} : (i, j) \in A\}$. If $c_{\min} < 0$, add $|c_{\min}|$ to the length each arc in the network so that they all become nonnegative. Then use Dijkstra's algorithm to solve the shortest path problem. Professor Wright claims that the optimal solution of the transformed problem is also an optimal solution of the original problem. [redacted]

   Give a small example showing that the claim is false.

4. Imagine you are navigating a ship, and you would like to get to port by using the least amount of fuel. You have a weather forecast, that gives you the direction and magnitude of the waves for each small grid square of ocean. For our problem we'll imagine that magnitude is the same everywhere, so that only the direction matters. Then, an imaginary forecast might look something like this:



   We have the forecast for the next three hours, split up in one hour time blocks. So, if the picture above was the forecast for the first hour, for the second hour, conditions may change to look like this:



Suppose that we also know the following fuel usage characteristics:

Wave direction	Fuel Usage per grid square
Same as direction of ship	0.5 units
45 degrees, but same direction as ship	1 unit
Perpendicular to direction of ship	2 unit
45 degrees, but opposite direction of ship	2.5 unit
Opposite direction of ship	3 units

We also know that it takes 15 minutes to traverse each grid square, regardless of the wave direction.

Formulate the problem of getting to port using the least amount of fuel as a shortest path problem. (Hint: First, how would you solve the problem if the forecast/wave conditions stay constant for all time? Once you have a solution for that, how would you incorporate a changing forecast?)

5. (AMO)

4.2. Beverly owns a vacation home in Cape Cod that she wishes to rent for the period May 1 to August 31. She has solicited a number of bids, each having the following form: the day the rental starts (a rental day starts at 3 P.M.), the day the rental ends (checkout time is noon), and the total amount of the bid (in dollars). Beverly wants to identify a selection of bids that would maximize her total revenue. Can you help her find the best bids to accept?

6. (AMO)

4.9. Personnel planning problem (Clark and Hastings [1977]). A construction company's work schedule on a certain site requires the following number of skilled personnel, called *steel erectors*, in the months of March through August:

Month	Mar.	Apr.	May	June	July	Aug.
Personnel	4	6	7	4	6	2

Personnel work at the site on the monthly basis. Suppose that three steel erectors are on the site in February and three steel erectors must be on site in September. The problem is to determine how many workers to have on site in each month in order to minimize costs, subject to the following conditions:

Transfer costs. Adding a worker to this site costs $100 per worker and redeploying a worker to another site costs $160.

Transfer rules. The company can transfer no more than three workers at the start of any month, and under a union agreement, it can redeploy no more than one-third of the current workers in any trade from a site at the end of any month.

Shortage time and overtime. The company incurs a cost of $200 per worker per month for having a surplus of steel erectors on site and a cost of $200 per worker per month for having a shortage of workers at the site [redacted]

Formulate this problem as a shortest path problem [redacted] (*Hint*: [redacted] use as many nodes for each month as the maximum possible number of steel erectors.)

7. You are in a foreign country where terrorists occasionally attack road convoys. You know the country's transportation network as a graph where roads (edges) connect cities (nodes). For every road, based on historical data, you know the probability that terrorists will attack a convoy traversing the road. For example, if there is a road between city i and city j, you know the probability an attack will occur on the road, p_{ij}. Attacks on each of the roads occur independently of one another.

   You have to move a convoy from city A to city B, using the country's road network. Describe how to find the safest, most reliable path connecting city A and city B. (Hint: Define the "reliability" of a path as the probability that a convoy using the path will not be attacked).

8. Create a homework problem on applications of shortest path or its algorithms. The problem you create shouldn't simply be computational, like "run Dijkstra on graph G," but instead something that cleverly applies or tests understanding of sortest path modeling or algorithms.

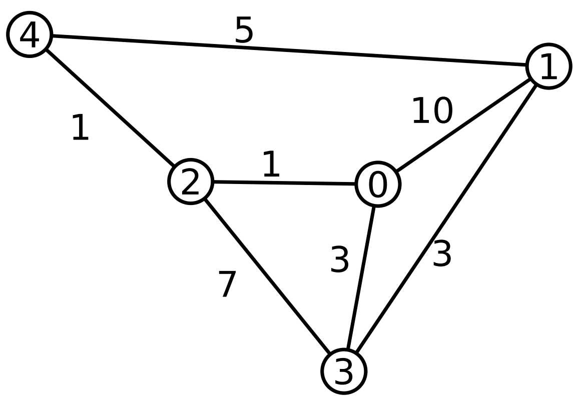


Figure 1: An example graph.