OA 4202, Homework 4

Nedialko B. Dimitrov

1. (AMO)

   6.32. Airline scheduling problem. An airline has p flight legs that it wishes to service by the fewest possible planes. To do so, it must determine the most efficient way to combine these legs into flight schedules. The starting time for flight i is a_i and the finishing time is b_i. The plane requires r_{ij} hours to return from the point of destination of flight i to the point of origin of flight j. Suggest a method for solving this problem.

2. (AMO)

   6.3. A commander is located at one node p in an undirected communication network G and his subordinates are located at nodes denoted by the set S. Let u_{ij} be the effort required to eliminate arc (i, j) from the network. The problem is to determine the minimal effort required to block all communications between the commander and his subordinates. How can you solve this problem in polynomial time?

3. (AMO)

   6.1. Dining problem. Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Show how to formulate finding a seating arrangement that meets this objective as a maximum flow problem. Assume that the dinner contingent has p families and that the ith family has $a(i)$ members. Also assume that q tables are available and that the jth table has a seating capacity of $b(j)$.

4. You are working for a group that has been monitoring a terrorist organization. Your group has intercepted enough communications and gathered enough intelligence to reconstruct the terrorist organization's social network... you know which terrorist is friends with which other terrorist. There is some possibility that this terrorist organization may soon split into two organizations. How would you find a likely split?

   For an alternate wording of the same problem, consider:

   6.11. Suppose that we wish to partition an undirected graph into two components with the minimum number of arcs between the components. How would you solve this problem?

5. You own your own temp-agency. Every month you have a pool of temporary workers and a pool of open jobs to be filled with temporary workers. Based on a worker's resume, you are able to discern the jobs the worker is eligible to fill. Specifically, worker i is eligible for the jobs in the set $N(i)$. Your agency receives \$5 for every feasible match between a worker and a job that you make. Of course, you can match each worker to at most one job, and each job

can be matched to at most one worker. How would you make the most pairings, to maximize your temp-agency's revenue?

6. (AMO)

6.2. Nurse staff scheduling (Khan and Lewis [1987]). To provide adequate medical service to its constituents at a reasonable cost, hospital administrators must constantly seek ways to hold staff levels as low as possible while maintaining sufficient staffing to provide satisfactory levels of health care. An urban hospital has three departments: the emergency room (department 1), the neonatal intensive care nursery (department 2), and the orthopedics (department 3). The hospital has three work shifts, each with different levels of necessary staffing for nurses. The hospital would like to identify the minimum number of nurses required to meet the following three constraints: (1) the hospital must allocate at least 13, 32, and 22 nurses to the three departments (over all shifts); (2) the hospital must assign at least 26, 24, and 19 nurses to the three shifts (over all departments); and (3) the minimum and maximum number of nurses allocated to each department in a specific shift must satisfy the following limits:

		Department 1	Department 2	Department 3
Shift	1	(6, 8)	(11, 12)	(7, 12)
	2	(4, 6)	(11, 12)	(7, 12)
	3	(2, 4)	(10, 12)	(5, 7)

Suggest a method using maximum flows to identify the minimum number of nurses required to satisfy all the constraints.

(Hint: While the way to solve this problem bears some resemblance to the tanker problem, the graph for this problem is *not* like the one for tanker problem we discussed in class. That is because each nurse can work at most one of the three shifts, and cannot simply continue from one shift to the next. If you need a further hint, see AMO Application 6.3.)

7. (AMO) You are in charge of a wood product company, and you would like to maximize the total wood yield of the forests owned by your company. Suppose your company controls two forests and wants to identify the best cutting schedule over a planning horizon of three years. Your company only harvests mature trees and studies performed by the company predict that w_ij wood units will mature in forest i on year j. If the wood is available for harvesting one year, but we choose not to harvest it, then it is also available for harvesting in the following years. In other words, trees grown up to harvesting size accrue over time. The company has also performed economic predictions that show it should harvest at least l_j wood units in year j. The economic predictions also show that if the company harvests more than u_j wood units in year j, the market would be flooded and wood prices would bottom out.

   Formulate the problem of determining a schedule with maximum wood yield as a network flow problem.